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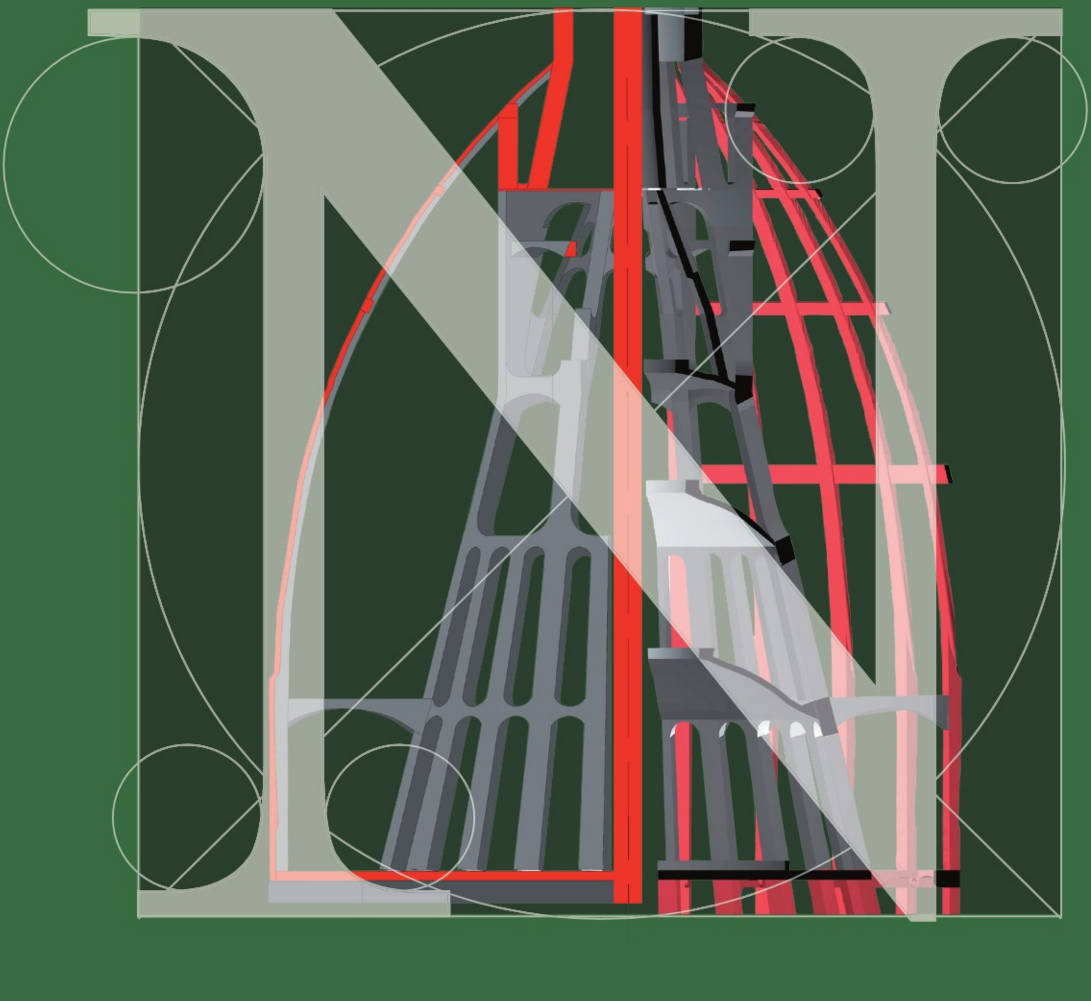

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Contents

The title of this issue of the *Nexus Network Journal*, "Architecture, Mathematics and Structure," is deliberately ambiguous. At first glance, it might seem to indicate the relationship between what buildings look like and how they stand up. This is indeed one aspect of what we are concerned with here. But on a deeper level, the fundamental concept of structure is what connects architecture to mathematics. Both architecture and mathematics are highly structured formal systems expressed through a symbolic language. In mathematics, the symbols are used to express precise mathematical concepts or operations, such as δ for derivative or ! for factorial. In architecture, the symbols may express function, such as a temple front for a court of law, or a dome for a church.

While the nature of the mathematical structure is unique, the nature of the structure governing an architecture design can vary. For instance, the generating structure can be geometrical, or musical, or modular, or fractal. Once we understand the nature of the structure underlying the design, we are able to "read" the meaning inherent in the architectural forms. The concept of structure is therefore a kind of Ariadne's thread for understanding architecture. The papers in this issue all explore themes of structure in different ways.

In "Continuity versus Discretization", Luisa Consiglieri and Victor Consiglieri examine how mathematical principles based on natural growth can be applied in architectural design to create a design structure that is dynamic rather than static. As they see it, it is the dynamic process of a cell *and its growth* that provides the basic structure. In support of their thesis, they examine the work of Peter Eisenman, Michael Hensel and Kas Oosterhuis, among others.

The use of pattern as underlying structure is addressed in "Cognitive-Mathematical Approaches for Evaluating Architectural Contextual Fit." The research team of Natheer Abu-Obeid, Fuad K. Malkawi, Khaled Nassar and Basel Al-eideh consider the relationship of a microstructure (a single building) to a macrostructure (the urban context) in order to determine the degree of "fit". They do this by means of establishing the architectural patterns that are characteristic of the two levels and then comparing them.

Another kind of pattern underlying macrostructure is found in the theory of chaos and fractals. This is studied at various levels by Mustapha Ben Hamouche in "Can Chaos Theory Explain Complexity in Urban Fabric?" To find the roots of the fractal-like growth of traditional Muslim settlements, relationships between urban development and Muslim law and traditions were studied.

Just as geometry provides an organizing structure at the scale of an entire building, it provides an organizing structure at the scale of ornament as well. It is no coincidence that all cultures in all epochs have turned to geometry to structure architectural decoration and ornament. In "Using Key Diagrams to Design and Construct Roman Geometric Mosaics?" Bernard Parzysz explains how given geometric patterns may have been distilled into key diagrams to facilitate their transmission and execution.

Geometry can also be used at the level of structural engineering as well. We need only think of the debate about whether a square or a triangle provided the most stable structure for the Cathedral of Milan. Massimo Corradi, Valentina Filemio and Massimo Trenetti have studied Alessandro Antonelli's beautiful structure in brick masonry for the dome of

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Novara's San Gaudenzio. In "Antonelli's Dome for San Gaudenzio: Geometry and Statics" they show how perfectly Antonelli understood the relationships between geometry and structure.

Sculptor Jesús Molina has undertaken a new study of the relationships between geometry and structure. In "The N4C Joint" Molina takes a structural principle that he developed in his sculptures to the scale of architecture. Readers of the *NNJ* will immediately recognize the similarities between the structures of Molina and those of Rinus Roelofs, also a sculptor who has recently published in the *NNJ* (see vol. 10, no. 1, 2008).

In "Nicola Zabaglia and the School of Practical Mechanics of the *Fabbrica* of St. Peter's in Rome", Nicoletta Marconi tells us about "structures for structures", that is, the various kinds of scaffolding designed and built by Roman master mason Nicola Zabaglia around the turn of the eighteenth century to facilitate construction and maintenance works.

Geometry is not the only provider of structure available to architects. Music has also made an important contribution to how architectural designs are structured. In "Music and Architecture: A Cross between Inspiration and Method", Alessandra Capanna analyses the work of three contemporary architects – Daniel Libeskind, Peter Cook and Steven Holl – to see what kind of structure music has supplied them with.

Another very important element that provides structure for architectural design is the module. In "Mathematics for/from Society: The Role of the Module in Modernizing Japanese Architectural Production, Izumi Kuroishi looks at how Japanese architecture were influenced by mid-twentieth-century European architecture as well as by the search for a national module that would modernize Japanese carpentry.

Sometimes geometry can be used as a device for organizing space. Geometry makes it possible not only to explain three-dimensional, but also to imagine higher dimensional space. Giovanni Ferrero, Celestina Cotti, Michela Rossi and Cecilia Tedeschi examine the relationship between geometry and space in "Geometries of Imaginary Space: Architectural Developments of the Concepts of M. C. Escher and Buckminster Fuller".

Also in the area of didactics this month, Laura Tedeschini Lalli interviews Maria Grazia Bartolini Bussi, co-author with Michela Maschietto of the book *Macchine matematiche.* The mathematical machines that Prof. Bartolini Bussi deals with have an architecture of their own, and her views of a laboratory for mathematics will interest teachers of mathematics and architecture.

This issue is completed by a conference report by Bettina Marten of the autumn 2008 conference in Dresden entitled "Fortifications in Focus", dedicated to the use of geometry to respond to the changing needs in defensive structures during the Renaissance.

I hope you enjoy reading this very rich issue of the *Nexus Network Journal.*

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Keywords: mappings (transformations), structures, structural engineering, patterns, Euclidean rhythm, morphogenetic rhythm, morphologic rhythm

Research

Continuity versus Discretization

Abstract. The threefold interest in architecture, biology and mathematics motivated us to examine and justify new architectural forms. We discuss some notions of rhythm: Euclidean, morphogenetic and morphologic. Contemporary relationships between structure and form are based on the generation of shape by technological processes, thus the resulting objects are restricted to their material expression. Here a phenomenological organisation of form and its continuity with the landscape arise out of the mathematical and architectural creativity. The use of the computer is applied from outside to inside instead from inside to outside; this means that we are dealing with the organisational processes via continuous methods instead of evolutionary processes given by computer simulations, known as *genetic algorithms*, where the resulting configurations are reduced to a matter of routine. The role of design as an aesthetic component innovates the theoretical framework of structural engineering to establish the architectural environments.

Introduction

Recent theories of physics and biology suggest the existence of organic forms in architecture [Alati, et. al. 2005; Harris 2007] in opposition to the static Greek theory. The present work promotes a rehabilitation of the concept of continuity as a requisite to find another system of theoretical mathematics in order to read the artistic values through emotion as an analysis of the sign. The present work promotes a rehabilitation of the concept of continuity as a requisite to find another system of eurhythmy. It introduces the notion of continuity as the basis of a new mathematical concept of the architectural form and of a new mathematical interpretation for the analysis of the sign, i.e., in order to read the artistic values of the objects. In fact, mathematics should not be only a technical tool for design science. It is in this sense that the alterations of the classical principles, which had been laid down in order to find the object's value (see below), and their relations with the continuity law should be determined.

Until today, the theories of configurations of forms were formulated following the geometrical principle of Plato and in harmony with the natural numbers. The concept of configuration was achieved through a rejection of dynamic movement. In fact, the Pythagorean School had established that all things were joined with a number without movement. Later Aristotle conceived a method for the knowledge of transformations, rejecting the mechanical movement once again. Thus, Greek science and philosophy have one major limitation: the rejection of the act of passing from one state to another as a rational explanation of the world. This Hellenistic idea obstructed the development of mathematics in the culture of architecture. There were two restraints: the incapacity to realize the concept of a function, and a disregard for the study of the natural phenomena.

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In spite of the rigorous study of the movement of the stars, essential for navigation at the end of the Middle Ages, the alteration of these principles was not as easy as it might seem, because the Greek philosophical spirit, established by the Platonic and Aristotelian systems, was deeply rooted. The mathematical tool for organizing images, relating space and shape, was based on the golden rectangle and its variants. The images were sets of families of one variable where the continuity between similar objects did not exist. We can understand the application of movement in architecture through the work of Klee [1961]. Since then, the laws of arithmetical and geometrical progressions could be overcome, leading us to examine the contribution of continuity and its properties to a new process of architectural composition. Moreover, to explain the value of an object in this context, we must to take into account how objects belonging the same family are related according to a law of growth. This interpretation, given by several theories (for instance, the phenomenological theory), becomes one of the objectives that must be applied in architecture. Bearing in mind that movement is an essential part of the growth of cells, the law of continuity can serve as a tool for the composition of architectural objects. It is in this perspective that the concept of Leibniz's infinitesimal and differential calculus can be applied to art. According to the concept of continuity, each object is a cell and its growth creates the final object [Consiglieri and Consiglieri 2006]. Thus, a new concept appears that substitutes the quantitative method with an analysis of continuous dynamics [Marchetti and Rossi 2006], which we will call *morphocontinuity*.

The outline of this work is as follows. First, we state three different notions of rhythm. The sections that follow are in accordance with aesthetic reasonings relative to the mathematical approaches under examination. The final section is devoted to some conclusions and open questions.

Notions of rhythm (mechanical versus organic)

Euclidean rhythm determines the characteristics of the final object by means of discrete modules arranged linearly and additively. No individual module can be changed directly in a continuous way in order to obtain an object, because it results from repetitions of its subparts proceeding to the next levels without continuous changes.

The morphogenetic rhythm is based on the concept of genes and the universal code of information in DNA, affirming the statement of Ch. von Ehrenfels at 1890 (see for instance [Guillaume 1937]) that the form as a whole is more than the sum of its elements. The impact of the theory of emergence in architecture demands new form-finding methods and a seamless integration to replace the conventional separation of design and material production [Weinstock 2006]. The process of self-organization consists of the algorithmic generation of a pattern according to local organizing principles at the cellular level. As opposed to Newtonian space (governed by the laws of mechanics), there arises what is designated as organic. Like Euclidean rhythm, organic space is made up of separate well-defined modules, and the final object is constituted by the dynamic connection between the parts, thus resulting in an open system.

Morphologic rhythm provides a mechanical interpretation, where the object shares the space and time as integrated elements and, basing itself on a continuous process, results in an architectonic state of equilibrium. We say that a cell C has a *morphologic rhythm* if

$$\exists R > 0, \quad C(t) \subseteq B_R(0), \qquad \forall t \in [0, T],$$

where $B_R(0)$ denotes an open ball centered at the origin (0,0,0) with radius R>0, and C represents a *cell* defined by

$$C := \{(x, y, z) \in IR^3 : f(x, y, z; t) \leq 1\}, \qquad (1)$$

with *x*, *y*, *z* representing (as usual) the spatial Cartesian coordinates and *f* denoting a continuous function defined in three-dimensional Euclidean space and depending on a time parameter $t \in [0,+\infty[$ (for details, see [Consiglieri and Consiglieri 2006]).

Serial discretization

A composition can be governed by different methods. The serial system creates a new code for the production of structures according to a recursive process, without necessitating the return to the generator module. The serial theory appeared as a technique for constructing new conceptual and structural realities and was only later consolidated into a theory. The form is not a pre-existent thing and a series becomes a mode of polyvalent method [Boulez 1975]. We can state that the serial discretization is an additive process.

To understand the present situation of how the series is an attempt to make the final objects more physically intimate with Nature, we refer to the works of Peter Eisenman, where are only considered sequences of sums of translations and rotations of generative elements. (fig. 1).

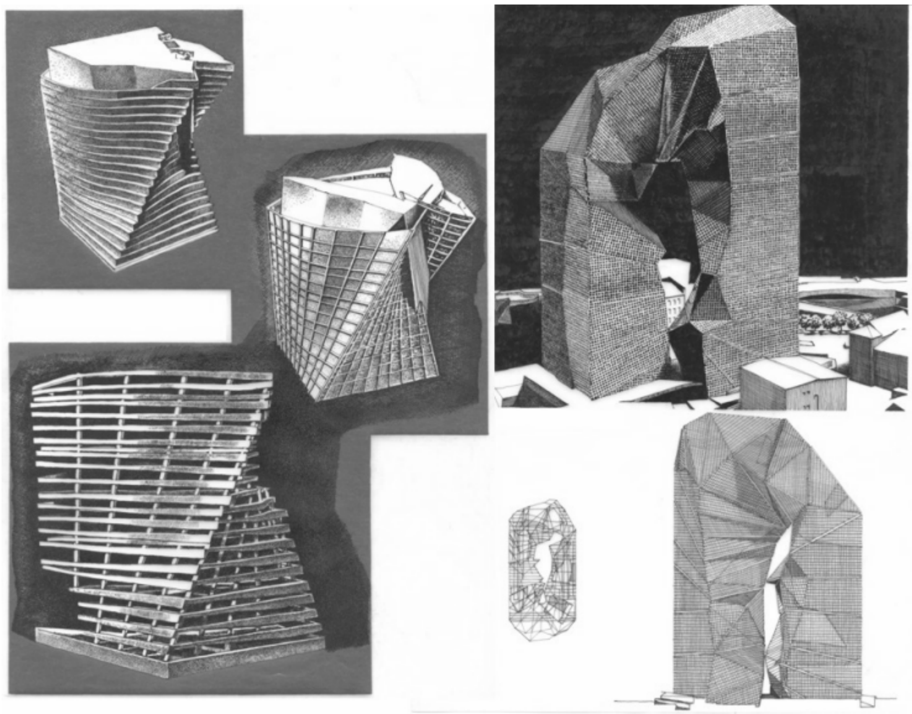

Fig. 1. Studies and Max Reinhardt Haus at Berlin by Peter Eisenman

Discretization of the theory of emergence

Generative theories change the concept of the shapes of an object in art. The creation of form can be processed through the integration of self-organization. By means of the interaction of each subpart of the whole, we obtain a unified image. For example, in fig. 2, a form embodies an "infinite number" lines. This concept can be found in the science of design [Kappraff 2001].

Thus the method of the periodicity is still totally accepted and interpreted as a philosophical act in accordance with a triangular scheme: beginning, middle and end.

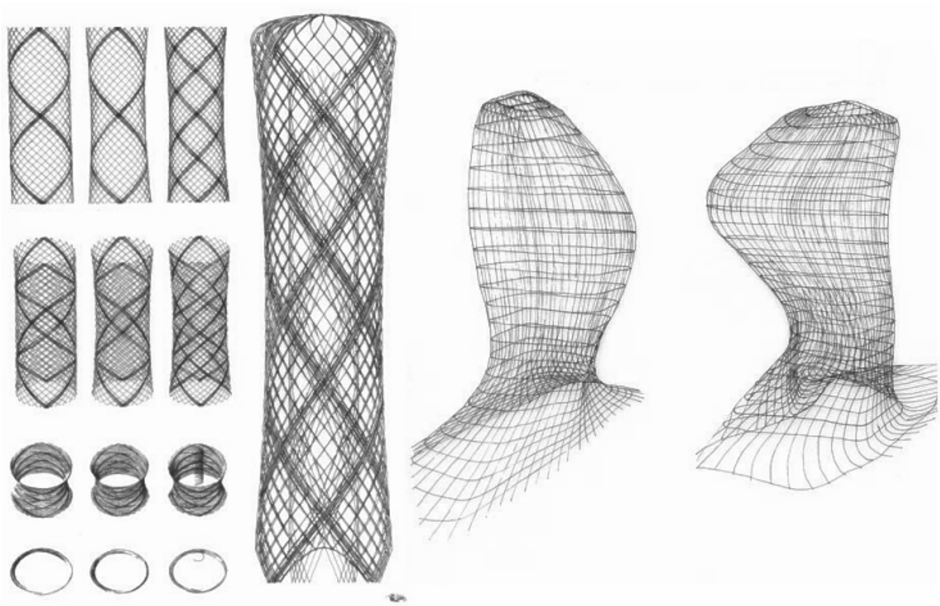

Fig. 2. Organic structural studies for cylindrical forms by Michael Hensel

Continuity of the forms (the negation of the number)

Let us explain the notion of morphocontinuity in the establishment of families of unities with different dimensions. We define a cell as in (1): a unit whose dimension is variable with time. In a phenomenological sense the cells allow, as whole, a modification in the specificity of we want to observe. The dimension is never represented by a number, but constitutes a variable within multiple architectonic aspects; in another words, a geometrical representation is a growing phenomenon that we must study in connection with its context. Thus, because we know the continuous properties of the mathematical functions, we can comprehend the variation of the analogies of the object. The object is a family of cells defined by continuous functions that depend on parameters of space and time becoming a dynamic object. This new concept establishes a new interpretation of form that till today belonged to the harmony of the numbers.

We define a *form* as the subpart of the boundary of the cell defined by

$$F := \{(x,y,z) \in A \times IR \subseteq IR^3 : \ z = f(x,y;t)\},$$

where f is a continuous function defined in *A* depending on a time parameter $t \in [0,+\infty]$.

Although the process of morphocontinuity is different from that of morphogenetics, the essence of both theories translates the final form in a 4D-environment (3D-space and 1D-time), which does not happen with Euclidean theories [Alati et.al 2005] (fig. 3).

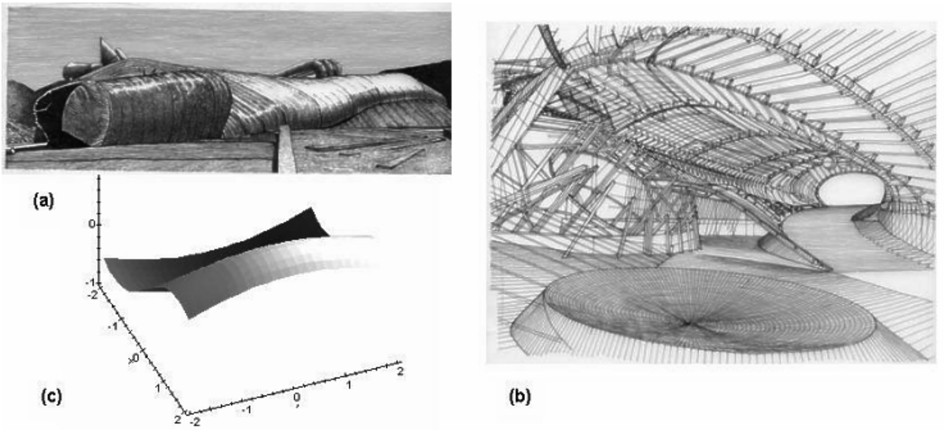

Fig. 3. (a) FresH2O pavilion (Netherlands, 1998) by Nox team and Kas Oosterhuis, (b) its discretization, (c) the graphical representation of a form under the continuous function $f : A = [-2,2]^2 \to IR$ defined as $f(x,y) = \sin(\pi x/3)\cos(\pi y/7)$. The graphic visualization is produced using Maple 4

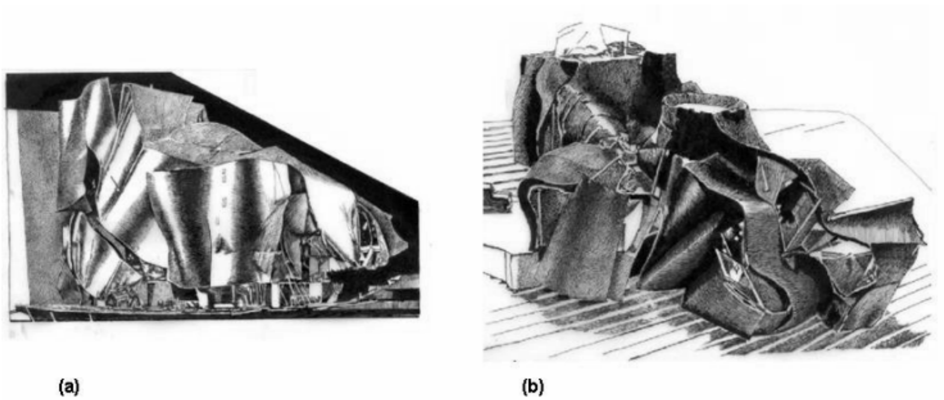

Fig. 4. (a) Modern Art Museum (South Korea, 1996), (b) Chiat House (Colorado, 1997), projects by Frank Gehry

In the struggle against Kant's *a priori* forms, which are known as ideal Euclidean forms of contemplation and that do not reflect real (sensible) space, the characteristics of Einstein's concept of space/time have been confusedly applied in the nexus between mathematics and architecture. The interactive cyberspaces for cultural activities in public buildings are a first step in comprehending the notion that time is an indispensable condition of our life and, consequently, is inseparable from space and from what exists outside us. For our purposes, a compartment exists in space-time where time is an independent entity.

In the works of Frank Gehry presented in fig. 4, the deformations of the cells serve as a basis for the systems of analogy of the form. The object is no longer a result of periodic schemes, and the image can be perceived through the sensations of the deformed shapes. The form is not the consequence of Euclidean geometry and its variants such as the serial theory, but is rather the consequence of the relationships of continuity that determine the structure of the unity. Gehry's geometrical forms also radically contest the morphogenetic process, although, structurally speaking, the construction of the form could have been made with an emergent cells patterns scheme.

Fig. 5 shows how by means of the continuous functions we can obtain dynamic effects of vibrant excitation without entering in disorder.

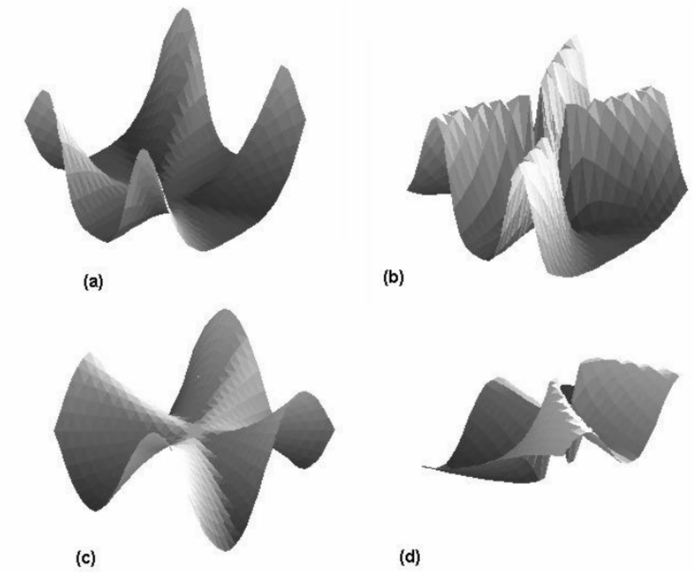

Fig. 5. The graphical representation of forms using $A = [-2,2]^2 \setminus \{(0,0)\}$, for the time parameter t>0 and $f(x,y;t) = \dfrac{x^4 y^4}{(x^2 + y^4)^t}$: (a) t=2, (b) t=3; and for the initial time parameter t=0: (c) $f(x,y) = \dfrac{x^3 y - x y^3}{x^2 + y^2}$, (d) $f(x,y) = \dfrac{x^2 y}{x^4 + y^2}$.
The graphic visualizations are produced using Maple 4

Singularities

As a consequence of the concept that the universe is matter in constant transformation, the regularities and the irregularities of the phenomena result in free will in the conception of architectural forms.

Differential calculus permits us to determine for each cell the relationships that correspond to singularities and sets of singular points. Then, the object is defined including dominant supporting points. For instance, the triangle has three singular points (the vertexes) whose expressive character should be taken advantage of in architectural practice.

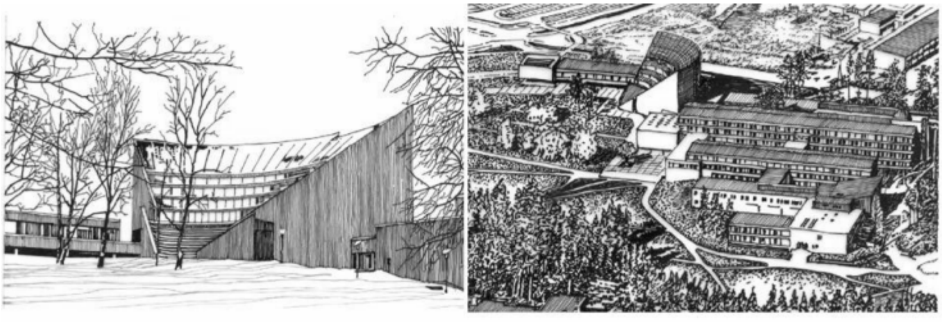

Fig. 6. Auditorium and Technological College (Finland) by Alvar Aalto

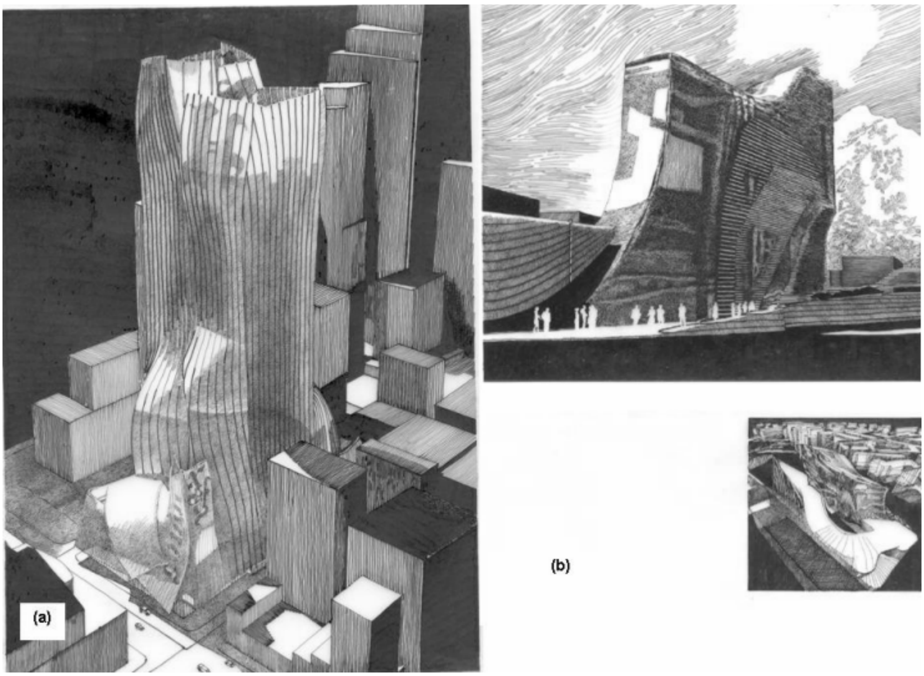

Fig. 7. (a) NY tower (USA) by Frank Gehry, (b) Euskotren tower (Spain) by Zaha Hadid

In Modernism this theme was applied in an essential way, where these representative singular points correspond to what we find in the works of Alvar Aalto (see fig. 6). On the one hand, there exist the border points between lateral surfaces and the roof that as a whole constitute an ensemble of singular points. On other hand, there exists a lower vertex that reveals an entry as a dominant supporting point. The object is a unified form that is harmonized through the curve constituted by the unique (non-differentiable) points, privileging the unique point of minimum height in the determination of the image. This point provides a sublime angle that heightens the impact of the surprise of the object on the principal façade.

The actual interpretation of the object is the result of the sensations and qualities of multiple images, that is, the form loses the principal façade. In fig. 7, the singular points are elements forming differentiable curves. These reflect the expression of tension [Kandinsky 1987] where the singularities are established by an exterior force.

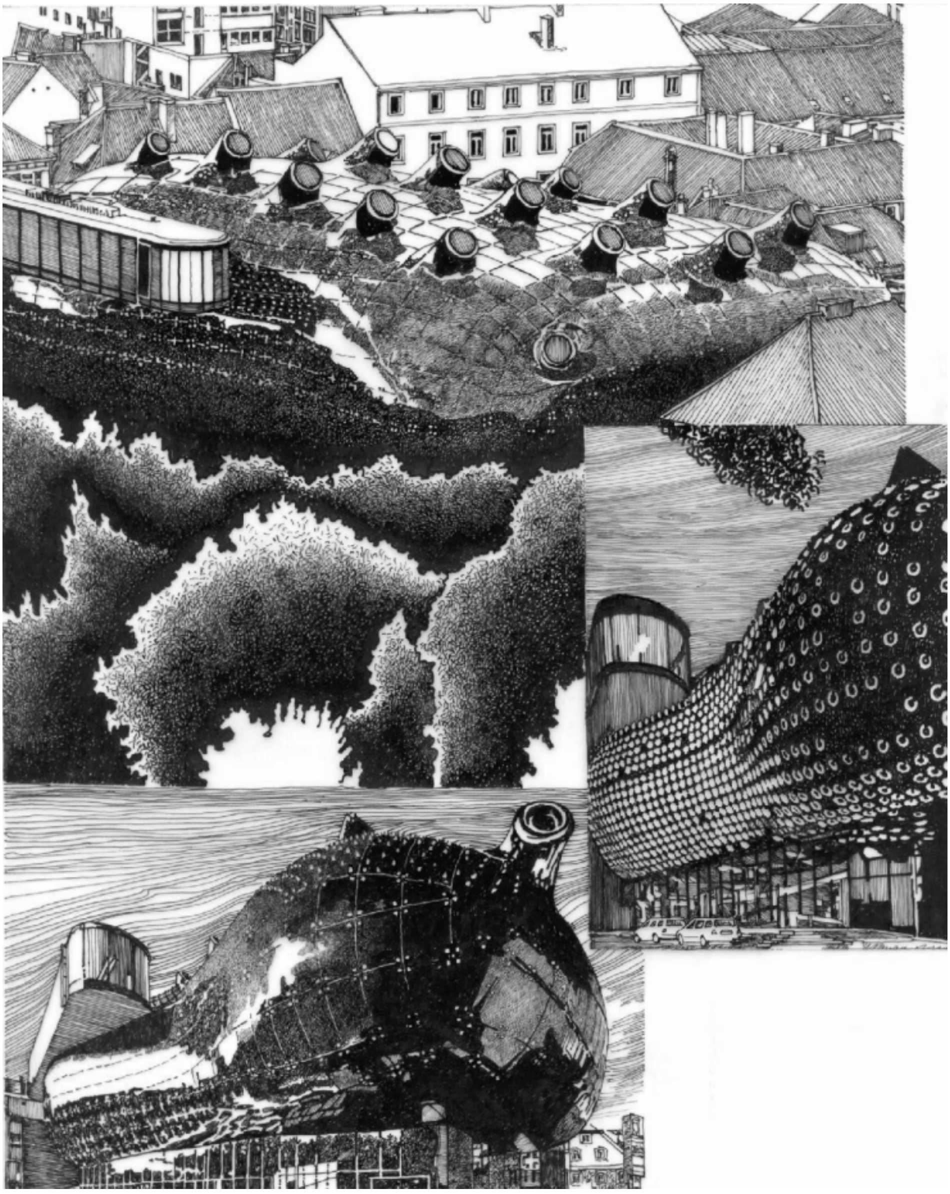

Fig. 8. Kunsthaus at Graz (Austria) by Jan and Tim Edler

Determination of the aesthetic value of the object (emotion and sensations)

When we analyze an object, in the first place we sense its characteristics, we perceive the rhythms of full and empty, clear and dark, color and texture, and so forth, and only afterwards do we become conscious of the value of that form. Sensations are considered the dominant elements in the determination of the images [V. Consiglieri 2007]. To comprehend objects we observe their shape, color, material and content. Thus these characteristics specify the object through a set of sensations and notions. These properties are not in the objects, but in ourselves. The emotion, as we observe it, organizes the objects in the architecture and the qualities of the objects that correspond to our senses. We conclude by saying that the aesthetic value is a consequence of the references given by the above properties and that it does not exist by itself, that is, the value of the objects exists in the sense of our spirit and do not have any reality outside this. Or simply, the object has no aesthetic value if humankind does not exist.

To understand the value of the mathematical curves referring to the objects that are shown in fig. 8, we should consider a scale of intensity for the sensations. Consider the scale of the sensations of the color vermillion, which is constituted by a variety of gradations going to their absence. When we study the notion of vermillion as a color, we study the successive (infinitesimal) segments of division, we compare them with other imperceptible tones of the same scale, and finally we verify the existence of the scale which is delineated such that it increases or diminishes the reality of the color. In the philosophical point of view, the emotion corresponds to the continuum (unity) of the sensations and it gives a unique value to an object. This new relationship between the elements of the objects is not determined by a number, as it was determined by the methods of the proportions, but by a variable. The aesthetic value is given to an object when we perceive the relationships between the curve and its tangent, which philosophically correspond to the character of the perception [Deleuze 1968], that is, the form that represents the desired object gives voice to its better perceptible quality as the distance between curves and its tangents tends to the intersection point. This principle represents a new concept for achieving beauty, like the golden number was used to achieve harmony in classical theories.

The linguistic essence

In linguistics, the substitution of a single letter can completely change the meaning of a word. In the case of the parallelepiped, the alteration of its length, height and depth can change its aesthetic value. Fig. 9 shows how images can be integrated through the positioning of small unities and their relationships in different cultural moments, namely, architectural rationalism (a), expressionism (b) and deconstructivism (c).

The studies connected with the grammar of a structural system may follow two approaches. First, the generative grammar corresponding to the discretization of the form pays attention to the sounds or letters that transform a material word into an abstract word, where the psychological process is not described. This recursive process in architecture formalizes an algorithmic strategy in the modeling of a pattern. Second, the grammatical method of Noam Chomsky corresponding to the continuity of the form, where the harmonic unity is not a word as it was for Ferdinand de Saussure and other linguists, but corresponds to the unity of phrase that represents a reasoning or proposition. This gives us a new pragmatism in the composition. Similarly, the architectonic composition can be analyzed through the execution of its form.

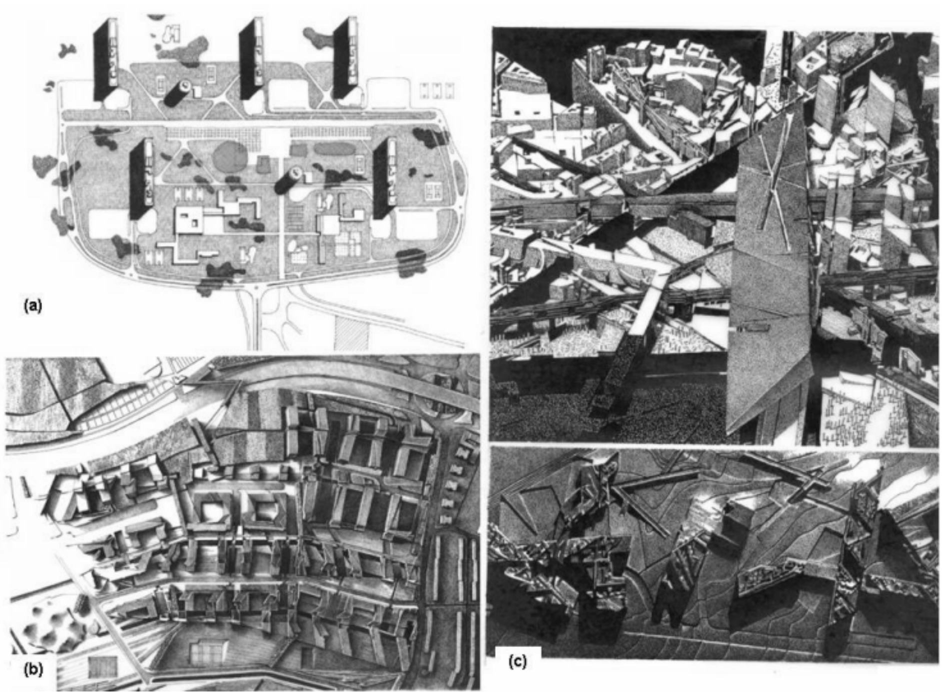

Fig. 9. (a) Garden City at Meaux by Le Corbusier, (b) Rebstock Park urbanism project by Peter Eisenman, (c) urbanism unities project by Daniel Libeskind

The message results from the meaning of the words and sentences and proposes, in the socialization process, a rationalization of the culture inherent to the panorama of the world. In the case of the house, considered as a proposition, it is an isolated house with one or more stories. It has recourse to a tautological character. While doing the architectonic analysis in the specification of the proposition and in the establishment of values, we have to consider the expression and the content. The content constitutes the combination of the spatial and formal elements in a rational sense and must have a useful value. The expression is neither a subjective attitude nor an individual feeling. It consists of what we give to the character of the image, a prerogative of critical reasoning, and constitutes a continuum of effects and of elements that belong to the structure of the object, entailing the form to a sign

Thus this tautology characterizes the knowledge of the proposition at the conceptual level of ideas, and the value of the propositions are interconnected to the symbolic codes, such as the functional typologies and the syntaxes that belong to each cultural moment.

The investigation of general configurations goes from images to signifiers with values. The forms are altered under a syntactical character, marking new physical and expressive aspects in the scales of proportions and light which introduce a real sensibility. In short, the emotion acts like a linguistic organization of logical and synchronic thought.

Conclusions

In the first place, we must recognize that the behavior of different forms depends on deterministic and indeterministic theoretical trends.

Following Heisenberg's Uncertainty Principle on which quantum physics is based, research into the manifestations of chaos interacts with the arts and other intellectual spheres [Mandelbrot 1982], according to the philosophical essay on probabilities.

To express mathematically the principle of determinism in accordance with the new laws of physics and biology, the principle of determinism introduced by Laplace must be redefined [Dahan-Dalmedico et al. 1992]. The new concept has to include the idea that any physical system is predictable, in the sense that at least after the previewed occurrence had happened, it is possible to verify that it was determined by the system state. This theoretical structure reflects an order that contains in its essence a fragmentation of the universe that gives us a perspective of its totality. In fact, it represents a primordial role on the new notion of determinism.

This present work is centered in this conflict between artistic principles, between the organic and the analytical. The object is presented as a cultural subject under different manifestations of human thought. We can synthesize the analytic approach proposed here in the following way:

First, the object has a life of its own during its existence, maintaining a constructive stability either in the internal spaces or through the external manifestations produced by natural accidents.

Secondly, the basis of the object stands in a physical structure determined by a state in which the supporting forces are organized, producing equilibrium defying the law of gravity.

The emotion of a form is perceived in three separate aspects: the visceral, the experimental and the behavioral. In spite of this, there is still no concise theory about emotion. There are recent works that try to address these problems, showing how in our world man comprehends things by means of the senses. The theories of emotions, whether from an individual or a general point of view, both start from a neutral intensity. One direction indicates what is not desirable, and from the same starting point, the other direction gives us the degree of pleasure of the relationship between the object and the observer. The method consists of a framework organized such that it attains a continuum of the different sensations. The values of the object appear as the result of the volumetric essences of surfaces, lines and elements, giving us formal comparisons in an ensemble of binary oppositions that is, between interior and exterior, vertical and horizontal, full and empty. So the formation of rational determinative relationships of the configurations is a theory whose main objectives is to make state precisely the concept of the form and to determine the efficacy of the positioning of the areas and volumes.

In conclusion, we are entering into two new theories, one of emergence and one of morphocontinuity, that correspond respectively to the perception of the infinite and the difference of the sensations, a possible step in the appreciation of the harmony that until today and for centuries was supported by the arithmetical and geometrical proportions with geometry as the only theory.

Acknowledgment

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Research

Cognitive-Mathematical Approaches for Evaluating Architectural Contextual Fit

Abstract. The main goal of this study is to apply a scientific quantitative approach to the investigation of contextual fit. This is approached mathematically within the framework of cognitive science and research on categorization and prototypes. Two experiments investigated two leading mathematical-cognitive approaches for explaining people's judgment of contextual fit of a new building with an architectural/urban context: prototype approach and feature frequency approach. The basic concept is that people represent the built environment via architectural prototypes and/or frequencies of encountered architectural features. In the first experiment, a group of twelve participants performed rank order tasks on artificially created architectural patterns, for the purpose of psychological scaling. Perceptual distances among all patterns were mathematically determined. In the second experiment, three groups of architectural patterns were constructed to represent assumed architectural contexts. The prototype of each context was mathematically determined according to prototype cognitive model, and based on the distances calculated in the first experiment. Fifty-six students participated in the main experiment, in which they rank ordered a group of fifteen architectural patterns in terms of contextual fit to each of the three architectural contexts. Participants' rank order data of the fifteen patterns were regressed on both the perceptual distances from prototypes, and numbers of features shared with each architectural context. Results indicated that both prototype and feature frequency approaches significantly accounted for important portions of participants' judgments. However, participants tended to prefer one approach to the other according to context composition. Results have implications for both research on utilizing cognitive-mathematical models in architectural research and on urban design guidelines and control.

Introduction

This study is based on the premise that understanding people's responses to architectural contexts requires knowing how they mentally represent the diverse forms of the experienced built environment as quantifiable images, which may be referred to as indices of contextual fit. The main goal of this study is to apply a scientific approach to the investigation of contextual fit. Investigating the contextual fit in the study is approached mathematically within the framework of cognitive science and research on categorization and prototypes. The basic assumption of the study is that people represent the diverse forms of the experienced built environment as abstract images, which account for the variation in its previously experienced exemplars and features. These abstract images could be either

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internally created mathematical averages of previous experiences in the form of prototypes, or a summation of the total frequency of shared features. The direct objective of this study is to evaluate the hypothesis that people mentally represent an architectural/urban context (group of architectural elevations) in the form of a prototype and/or a feature frequency image, according to which they judge the belongingness of novel (infill) patterns to that specific architectural /urban context.

The need for this research

There has been a growing public dissatisfaction with the visual impact of environment in the past few decades [Sanoff 1991]. Accordingly, many countries began developing and applying new legislations that require solid assessment of environmental aesthetics. Research indicated that the public aesthetic dissatisfaction with their environments is associated with the differences between professionals and the public in their environmental judgments and preferences [Canter 1969; Groat 1982; Groat 1984]. It is also argued that this dissatisfaction is a result of the lack of empirical studies that validate urban design controls [Stamps 1995]. Developing public policies for aesthetic controls requires community support by involving people in shaping their visual environment [Sanoff 1991]. In this context, Stamps [1995] suggested "conducting scientific experiments on public preferences before the regulations are implemented". Along this line, Stamps [2000] has conducted a considerable number of empirical studies and experiments in an attempt to apply a logical solid theory of environmental aesthetics, which also proposed to measure aesthetic impact. Within this context, the present study investigates another approach to evaluate architectural contextual fit quantitatively based on people's perceptions.

Architecture influences the perception, memory, and clarity of the images that people form about their environments. Within this context, environmental ambiguity and imageability can be better understood by the perception of contextual fit. The discussion of contextual fit has traditionally ranged from philosophical perspectives to physical design tactics and cues. Harrison and Howard [1980] considered the physical components of the environment necessary to evoke strong images in observers. Appearance, which is an important aspect of the physical components, included shape, pattern, form, color, size, material, and design (pattern).

Theoretical background

Pattern recognition theories

Our environmental knowledge develops both from the regularities in the environment and the difference between the distinctive examples and the knowledge structure [Purcell & Nasar 1992]. Several psychological theories and models, including quantitative ones, have been explored to explain our understanding of the environment. Categorization and prototypicality studies have provided us with many of these models. Most theories of categorization are based on the idea that all instances of a category share something in common and that this "something", however defined, is necessary for a particular instance to belong to a category [Kumatsu 1992]. Many of the previous theories followed one of two major streams: template-matching and feature analysis [Gibson 1969; Neisser 1967; Reed 1972; Reed 1978]. Feature analysis proposes that the process of pattern recognition includes decomposing the pattern into its constituent features and comparing these features with the stored ones. Template matching proposes that "an unknown pattern is represented as an unanalyzed whole and is identified based on its degree of overlap with various standards" [Reed 1978: 139]. Feature analysis theory is generally more flexible than template matching theory, though it has the problem of configuration. For example, the same features could be used to construct many unrecognized and even scrambled patterns. On the other hand, one major problem of template matching theory is its incapacity to account for pattern variability and transformation. In response to the above criticisms of both theories, and based on the logic of exemplar variety and associated typicality, a prototype theory evolved.

The basic idea of the prototype theory is that the variety in exemplar differences from the generic knowledge creates experiences that vary in consistency with the knowledge structure. According to Rosch and Mervis [1975], members of a category regarded as typical shared more features with other members of the category than with non-typical members. In other words, examples that are consistent with the knowledge structure are judged as typical of knowledge structure. These typical examples were termed "good examples" by Rosch and Mervis [1975] and by Purcell [1984, 1986]. Prototype theory assumes that people establish a "virtual" member to represent the average member of a certain category. Using random configurations of dots, Posner and Keele [1968] demonstrated that participants abstract category prototypes and subsequently use them for classifying novel patterns. They found that the patterns most distant from the prototype were the most difficult to recognize. To account for these data, a prototype distance model was proposed by Posner [1969]. According to Posner's model, when a series of similar visual stimuli are experienced, a prototype is created at the point in the multidimensional similarity structure that represents the greatest similarity to all stimuli. Using geometric forms which were systematic analytic transformations of a prototype, Franks and Bransford [1971] demonstrated that participants could construct a prototype to represent the central tendency of patterns. Their experiment consisted of an acquisition phase and a recognition phase. During the acquisition phase, participants were exposed to a subset of the instances of a pattern that varied in their transformational distance from the prototype. The recognition phase consisted of the prototype and a subset of the instances of a pattern that varied in their transformational distance from the prototype. Recognition ratings were found inversely related to an item's transformational distance from the prototype, with the prototype receiving the highest recognition rating. This study was further developed by Abu-Obeid and Tassinary [1993] using schematic architectural patterns. However the psychological distances among these patterns were empirically determined according to the model of Posner [1969]. The findings of Abu-Obeid and Tassinary confirmed the earlier findings of Franks and Bransford, as people's recognition of novel architectural patterns was found to be an inverse function of the transformational distances from prototypical elevations.

Previous studies have also compared the feature and prototype models as explanatory accounts of people's patterns recognition. Generally speaking, these studies demonstrated that people tend to follow prototype models when recognizing human faces, while they tend to follow feature frequency model when recognizing geometric patterns. Using schematic human faces, Reed [1972] found that more study participants used prototype strategy than feature frequency strategy in their judgments. Neumann [1977] found that people follow a feature frequency approach when processing geometric patterns, yet follow a prototype approach when processing human faces.

Levels of affect and other types of experience were found to be associated with the levels of typicality to the knowledge structure of the encountered examples [Mandler 1984; Purcell 1984]. Purcell [1986] reported the occurrence of an affective response when there is a discrepancy between a current instance of a church building and its typical examples in memory. Purcell & Nasar [1992] related typicality to style and associated typicality with the experience of familiarity. Typical examples are judged as familiar, while examples that are discrepant from the knowledge structure are judged as atypical and experienced as unfamiliar.

Method

Previous studies on pattern recognition investigated two leading and competing approaches, which were argued to provide an explanation for people's recognition of novel patterns: 1) the prototype approach and 2) the feature frequency approach. The experiments reported below were conducted to determine whether people represent a group of related architectural forms (e.g., architectural context: street façade) via architectural prototypes and/or frequencies of encountered architectural features when judging an added infill, or judging the fit of a new building to an old context.

Study scheme

By manipulating specific design variables, a total set of artificial architectural patterns was to be created for use as architectural stimuli.

The study had to introduce hypothetical architectural contexts by creating examples of street façades, which were to be constructed from previously created set of architectural patterns. Cognitively speaking, these street façades were to be used as reference learning patterns.

A subgroup of architectural patterns was to be selected from the previously created total set of architectural patterns. This subgroup was to be introduced as possible infill to the street façades. Cognitively speaking, these infill patterns were to be used as reference recognition patterns.

A prototype pattern had to be identified for each street façade. This was achieved by applying multidimensional scaling experiments, through which dissimilarity judgments among all patterns of the previously created total set of patterns were transformed into perceptual (dissimilarity) distances. A prototype of a street façade is the pattern which has the least accumulated perceptual (dissimilarity) distance from all other patterns in the façade.

Prototype theory argues that the fit of any recognition (infill) pattern with the street façade is judged as a function of its perceptual distance from the prototype of that façade. This distance should be identified using multidimensional scaling.

In addition to its distance from prototype, each recognition pattern has a number of shared features with the patterns of the street façade (feature frequency). The feature frequency theory argues that any novel pattern is judged in its fit with the street façade, according to its accumulated number of shared features with all patterns of the street façade (feature frequency).

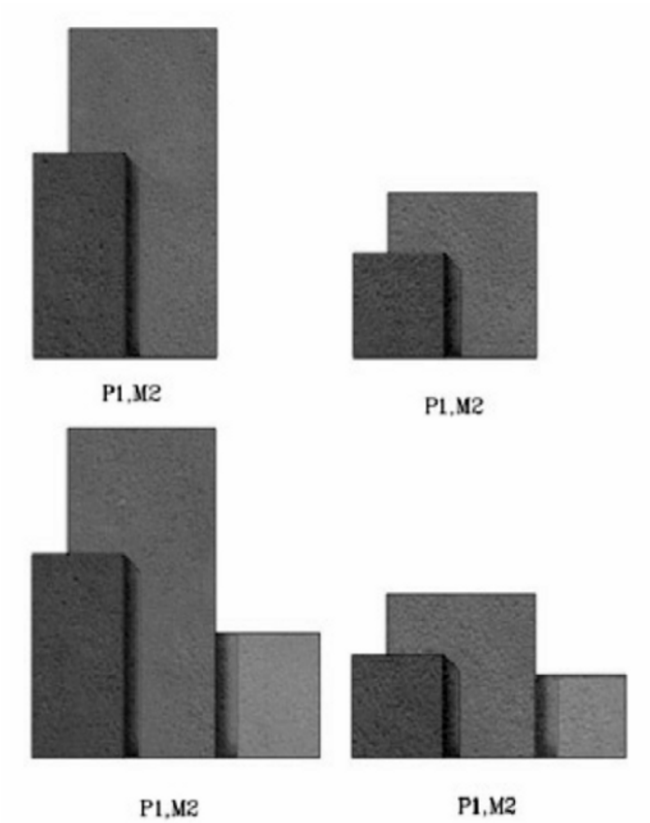


Fig. 1. Alternatives based on number of masses and proportions

In a recognition experiment, subjects were to judge a group of recognition patterns in terms of their fit with a street façade. Their judgments were to be statistically tested as a function of the patterns' distances from prototype and/or their features frequencies.

Stimulus material: contextual cues and design variables

Visual properties of buildings are to be addressed as the contextual variables in this study. According to Habe [1989], more than 98% of design controls are on visual properties of buildings. In a study by Groat [1983], she specified space, massing and style as aspects of design strategy according to which architects may fit the new to the old. In another study, Groat [1988] investigated the design attributes of contextual compatibility. She found that the façade components had the strongest effect on compatibility judgment. Bentley et al. [1985] suggested several contextual cues, such as windows, doors, and wall details as reference variables for contextual fit. Generally speaking, previous studies indicate that judgments of buildings are related primarily to building façades. Stamps [1995] presented a method to validate contextual urban design principles as a design control. In his study, Stamps identified two reference design variables: scale and character, according to which a new building could be matched with a group of older ones.

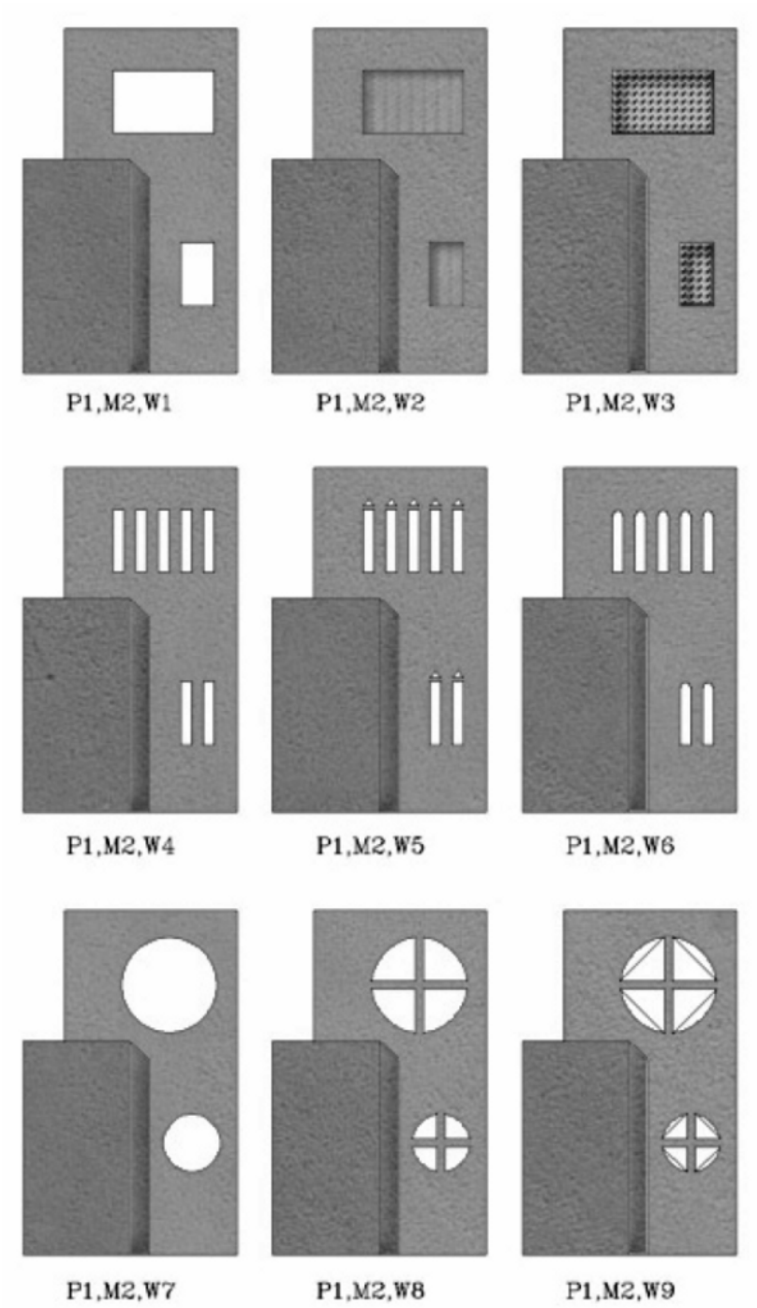


Fig. 2. Alternatives based on window form

In another study on preferences for residential façades, Stamps [1999] identified three factors of architectural façades: surface complexity, silhouette complexity, and façade articulation.

The present study builds upon the previously mentioned reference variables. Given the sheer number of possible combinations, it would be complicated to manipulate a large number of design elements in a single study systematically. Consequently, this study used simple artificial architectural elevations that were varied systematically by manipulating three design elements. In addition to proportion, this study identified the number of masses and the form of windows as contextual design variables. The elevations created were the result of applying combined transformational possibilities to the three design elements: 1) the number of masses (two possibilities – fig. 1); 2) the proportion of masses (two possibilities – fig. 1); and 3) the form of windows (nine possibilities – fig. 2), making a total set of thirty-six alternative elevations (2x2x9). The final set of all elevations was drawn with simple rendering as shown in Appendix I.

Experiment 1: stimuli scaling

The main objective of this experiment was to determine the perceptual distances of dissimilarities among all the stimuli which were to be used as part of the independent variables in Experiment 2.

Participants

Twelve undergraduate university students (seven males and five females) volunteered to participate. The participants were neither architecture nor art majors.

Procedure

Participants, one at a time, were presented with the stimuli, which consisted in the total set of thirty-six patterns. They were asked to rank order the total set of patterns in terms of similarity to a single reference pattern, which was to be randomly chosen from the total set and rotated for each trial by the experimenter. The participant rank ordered all the elevations of the total set in terms of similarity using a rotating reference technique. In each trial, the individual reference pattern was selected by the experimenter and was recorded in a matrix of rank decisions taken by the participant. Then, the participant rank ordered the rest of the total set (35 patterns) in terms of similarity to the single reference pattern. This was considered as one trial (ranking task). A total rotation of thirty-six reference patterns produced a matrix of ranks (36x36) for each participant. To complete this matrix, each individual had to participate in thirty-six trials. The order of presenting the reference patterns was randomized between participants. The order of distributing the to-be-ranked patterns in front of the participants was randomized both within and between participants. The experimental session took approximately ninety minutes per participant.

Analysis and results

The resulting data of experiment 1, hereafter referred to as proximity data, were in the form of square asymmetric matrices of dissimilarities between patterns of the total set. Each matrix represented the responses of one participant. A multidimensional scaling analysis was applied to the collected data in the total set of patterns (thirty-six) to determine the relative perceptual dissimilarity distances among all the patterns in the set. The thirty-six patterns of the total set were spatially conceptualized as scattered along a visual space, which has a specific number of visual dimensions. Each pattern was represented by a projection on each visual dimension (a detailed mathematical explanation is provided in Appendix II). The number of dimensions for the total set of patterns was determined according to the lowest badness-of-fit (the best model fit) criterion. Scree analyses were performed and showed that a space of four-dimensional solution best fit the patterns, which was adopted as a reference scale for the main experiment (Experiment 2).

The first objective of the above analyses was to measure the perceptual (dissimilarity) distances among all patterns, which represent the levels of their dissimilarities. Determining such distances would facilitate determining the central representation (prototype) of any selected group (category) of elevations from within the total set. It would also facilitate determining the closeness of each recognition elevation to the prototype. Calculation of all possible distances between all patterns for the total set (thirty-six patterns) was performed using the Euclidean distance model. The final product of these calculations was a matrix comprised of all possible Euclidean distances between all patterns, which resulted in 630 possible distances.

Experiment 2 (Main Experiment)

The purpose of this experiment is to evaluate the hypothesis that people mentally represent a group of architectural elevations (architectural/urban context, street façade) in the form of a prototype and/or a feature frequency image, according to which they judge the belongingness of other novel (infill) patterns to the context.

Research participants

Fifty-six undergraduate university students (thirty males and twenty-six females) participated in this experiment. None of the participants were students of architecture or of art majors.

Stimulus material

Context groups (architectural contexts of street façades): Three groups (GA, GB & GC) of the previously created architectural elevations were constructed as reference architectural/urban contexts (street façades). Each group was composed of five different elevations to represent a selected portion of an assumed street façade, which in its turn represented an architectural context (fig. 2). The reason for using three context groups in the experiment instead of only one group was to investigate and reveal any effect of specific group compositions. A prototype pattern was determined to represent each of the three context groups. The selection of this prototype was based on the previously mentioned principle of central tendency [Posner 1969; Posner & Keele 1968]. In mathematical terms, the prototype pattern was selected as having the least accumulated perceptual dissimilarity distances from all of the five elevations of its associated context group. The perceptual distances were calculated according to the Euclidean distance model. To find this prototype, the total set of thirty-six patterns was subject to mathematical evaluation, through which the pattern having the least sum of the distances from its associated context group (five patterns) was to be specified as the group prototype. These distances were previously determined as a result of experiment 1. The mathematical calculation of prototypes revealed the existence of multiple prototypes for all groups. To make the study less complex, only one of these prototypes was selected for each context group for further analysis. This was done in a follow up session to experiment 1. Each participant in experiment 1 was invited again to this session and was presented with all prototypes for each context group separately. The participant was then asked to select only one prototype that he/she thought was the most representative of its context group. The most frequently selected prototype by participants for each context group was then used in the main experiment as the main prototype of its context group (fig. 3).

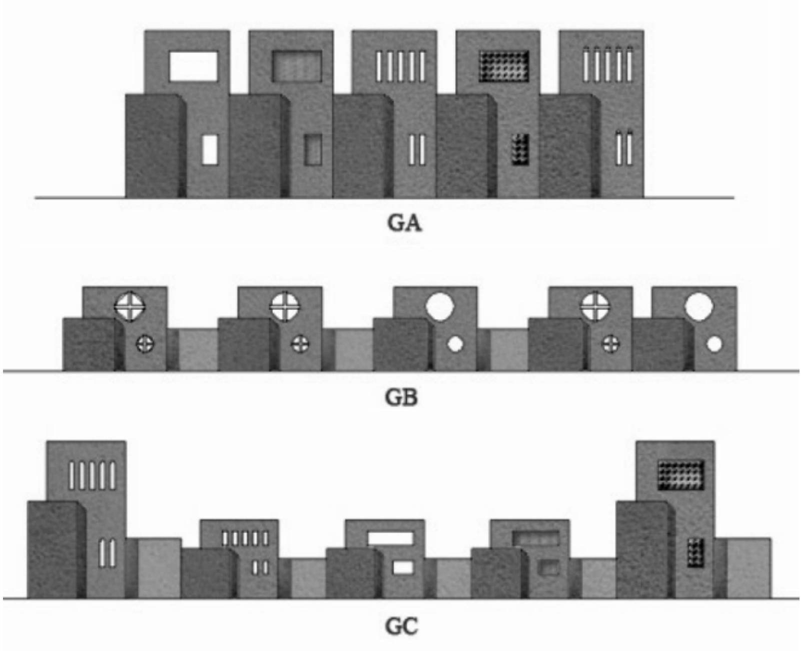


Fig. 3. The three context groups of patterns, which represent street façades (architectural contexts)

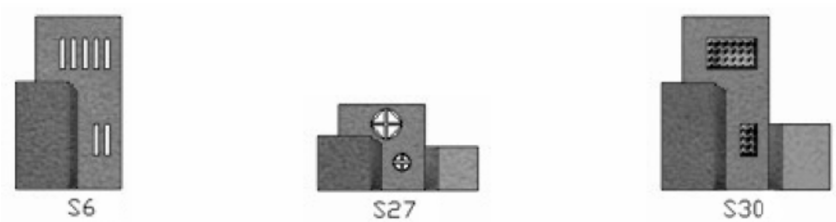


Fig. 4. The selected prototypes for the three context groups (S6 is the prototype for Group A, S27 is the prototype for Group 3, and S30 is the prototype for Group C)

Infill group: A group of fifteen patterns (architectural elevations) was selected by the experimenter as an infill group. Each of these patterns was considered as a possible new addition to each of the context groups (street façades). The infill patterns were rank ordered by each participant according to their fit with each one of the context groups (street façades). The selection of the infill group took into consideration the diversity of elevation compositions and design features and the diversity of psychological distances from reference groups' prototypes. The three selected prototypes were included in the infill patterns group (Table 1).

Infill Pattern	Pr: Perceptual Distances from Prototype			Fr: Feature Frequencies (Total Number of Shared Features with Context Group)		
	Group A	Group B	Group C	Group A	Group B	Group C
S1	0.14	2.57	0.80	17	0	4
S2	0 .02	2.69	0.92	17	2	8
S4	0.28	2.99	1.22	15	0	6
S6*	0 .00	2.77	0.94	17	0	4
S12	0 .07	2.78	1.01	2	2	2
S19	1.02	1.69	0.08	2	13	11
S21	1.02	1.69	0.08	2	13	11
S22	1.23	1.48	0.29	0	13	14
S25	2.61	0.10	1.67	0	17	9
S27**	2.71	0.00	1.77	0	15	9
S30 ***	0.94	1.77	0.00	12	3	8
S31	1.27	1.44	0.33	10	3	8
S34	2.53	0.18	1.59	10	7	6
S35	2.63	0.08	1.69	10	7	6
S36	2.55	0.16	1.61	5	5	6

* prototype of group A ** prototype of group B *** prototype of group C

Table1. Independent variables: Values of perceptual distances from prototypes and total frequencies for all recognition stimuli on three context groups

Independent variables

Each of the infill patterns was assigned two mathematical values in relation to each reference group:

- Its calculated perceptual distance from the context group prototype. This is according to the Euclidean distance model.
- The total frequencies of shared features with the patterns of each of the context groups.

These two values were considered as two independent variables in the analysis to follow. By considering the three context groups, six mathematical values (2x3) were then to be assigned for each infill pattern (Table 1).

Dependent measure

The fifteen infill patterns were rank ordered in terms of fit with each of the three context groups. The study considered that the primary goal of each participant's ranking is to select the infill pattern with the best fit (rank 1). Then percentages of the number of participants assigning rank 1 for each infill pattern were treated as dependent measures. In other words, for each infill pattern, there was a percentage of participants who considered it the best contextual fit with the context group. This percentage was calculated and treated as a dependent measure for that infill pattern and in reference to a specific context group. By considering the three context groups, each infill pattern should have three dependent measures (Table 2).

Infill Patterns	Percentage of Frequency of Best Rank (1) for Each Infill Pattern		
	Group A	Group B	Group C
S1	19.6	1.8	1.8
S2	12.5	1.8	1.8
S4	23.2	1.8	5.4
S6*	30.4	0	3.6
S12	3.6	7.1	1.8
S19	1.8	0	12.5
S21	0	0	5.4
S22	0	0	10.7
S25	1.8	33.9	0
S27**	1.8	44.6	1.8
S30***	3.6	0	33.9
S31	1.8	0	21.4
S34	0	0	0
S35	0	5.4	0
S36	0	5.4	0

* prototype of group A   ** prototype of group B   *** prototype of group C

Table 2. Dependent measures: Percentages of frequencies for rank one (best contextual fit)

Procedure

Participants, one at a time, were presented with only one reference group of stimuli (five patterns), which were assumed to represent a portion of a street façade and were to be used as a reference group in a recognition task. With each of the three context groups, the participant was simultaneously presented with a series of fifteen infill patterns that were to be rank ordered in terms of the best fit with the presented context group. The subject was asked to assume that the presented group of infill patterns was going to be rank ordered in terms of its contextual fit with the presented street façade (context group), where rank 1 in the evaluation scale represented the best contextual fit with the context group, and rank fifteen represented the lowest contextual fit with the context group. The rank ordering task was performed by each participant for each of the three context groups. This means forty-five (15x3) ranking judgments were made by each participant. The order of presenting the fifteen infill patterns with each reference group was randomized among the participants, and the order of presenting the context groups to the participants was rotated among the participants. The experimental session took approximately forty-five minutes per participant.

Analysis and results

Both simple and multiple linear regression analyses were applied to the percentages of best rank (rank 1) for infill patterns as related to both distances from prototypes and feature frequencies on each of the three context groups (GA, GB & GC). Data and analyses were characterized as follows (Table 3):

	Regression Models According to Context Groups & Independent Variables Included in Each Model								
	Group A			Group B			Group C		
	Mod.1 *Pr*-a, *Fr*-a	Mod.2 *Pr*-a	Mod.3 *Fr*-a	Mod.1 *Pr*-b, *Fr*-b	Mod.2 *Pr*-b	Mod.3 *Fr*-b	Mod.1 *Pr*-c, *Fr*-c	Mod.2 *Pr*-c	Mod.3 *Fr*-c
R	0.816	0.674	0.740	0.607	0.525	0.571	0.729	0.729	0.310
R^2	0.666	0.454	0.548	0.369	0.276	0.326	0.531	0.531	0.096
Adjusted R^2	0.610	0.412	0.513	0.263	0.220	0.275	0.453	0.495	0.027
Model Sig.	0.001*	0.006*	0.002*	0.063	0.044*	0.026*	0.011*	0.002*	0.261
Pr Sig.	0.062			0.388			0.006*		
Fr Sig.	0.017*			0.209			0.993		
B Coef. *Pr*	-3.709	-6.254		-3.212	-6.279		-10.79	-10.78	
B Coef. *Fr*	0.795		1.098	0.899		1.290	-0.006		0.957

*significant ($p<0.05$)

Table 3. Regression analysis for percentages of rank 1 rates

Dependent measures of the study were the percentages of first ranks for the fifteen infill patterns for each context group and for the fifty-six participants.

Independent variables of the study were two:

1. Dissimilarity distance between each infill pattern and the prototype of each context group (*Pr*);
2. The frequency of shared features between each infill pattern and each context group (*Fr*).

Regression analysis included three models:

1. Model 1 is a multiple regression, which included the two independent variables (*Pr & Fr*);
2. Model 2 is a simple regression, which only included the independent variable (*Pr*);
3. Model 3 is a simple regression, which only included the independent variable (*Fr*).

The results demonstrated the following:

In the case of context group A (GA):

- Multiple regression model 1, which included both independent variables (*Pr*-a & *Fr*-a) turned out to be highly significant: ($p=0.001$). $R^2 = 0.666$;
- Simple regression model 2, which only included (*Pr*-a) was found statistically highly significant: ($p= 0.006$). $R^2 = 0.454$;
- Simple regression model 3, which only included (*Fr*-a) was found statistically highly significant: ($p= 0.001$). $R^2 = 0.548$.

In the case of context group B (GB):

- Multiple regression model 1, which included both independent variables (*Pr*-b & *Fr*-b) turned out to be almost significant: (p= 0.063). R^2 = 0.0.369;
- Simple regression model 2, which only included (*Pr*-b) was found statistically significant: (p= 0.044). R^2 = 0.276;
- Simple regression model 3, which only included (*Fr*-b) was found statistically significant: (p= 0.026). R^2 = 0.326.

In the case of context group C (GC):

- Multiple regression model 1, which included both independent variables *(Pr*-c & *Fr*-c) turned out to be significant: (p=0.011). R^2 = 0.531;
- Simple regression model 2, which only included (*Pr*-c) was found statistically highly significant: (p= 0.002). R^2 = 0.531;
- Simple regression model 3, which only included (*Fr*-c) was found statistically not significant: (p= 0.261). R^2 = 0.096.

Discussion

A general reading of the results shows the following:

- The two independent variables (*Pr* & *Fr*) explained considerable percentages of the data variance for the three street façades;
- There is an insignificant improvement in the model performance when the two independent variables are included as compared to only including one of them, except for the case of context group C;
- Regression and significance values for different regression models slightly vary across the three street façades, except for the case of context group C.

The results indicate that either distance from prototype or feature frequency accounted for a significant portion of participants' judgments of best contextual fit for the infill patterns. It seems that both strategies are probably followed by people in judging contextual fit. Regression analysis demonstrated that in the case of prototype approach, the more the infill patterns were distant from the prototype, the less the participants judged them to belong to the context groups. This finding is in support of previous studies on prototype approach [Posner 1969; Franks & Bransford 1971; Abu-Obeid & Tassinary 1993]. In the case of feature frequency approach, the more features infill patterns shared with the context groups, the more the participants judged them to belong to the context groups. This finding is in support of previous studies on feature frequency approach [Reed 1972; Neumann 1977]. This tells us that what the participants had learned about architectural contexts was not limited to the specific instances of the context group. It seems that the participants acquired general information about the whole group during scrutinizing. This general information appears to be represented by both the group's prototype and an accumulation of features. Moreover, the multiple regression models seem to be performing a little better than the simple regression models as they have higher R^2 values, except for street Group C, in which multiple regression model and prototype model (*Pr*-c) have the same R^2 value and the prototype model is higher in significance. In the case of group C, R^2 value for *Pr* (distance from prototype) was significantly higher than that of *Fr* (feature frequency). As a matter of fact, the simple regression model for feature frequency (*Fr*-c) was not statistically significant (p=0.261). It seemed that participants tend to favor a distance from prototype strategy when referring to context C. This may reinforce the probability that participants employ double strategies in their judgments of some street contexts (as in the case of groups A and B) and employ prototype strategy in others (as in the case of group C). Looking at the composition of the three street groups, we notice that groups A and B are more harmonious in terms of proportion than street group C, which has the two height proportions. It is possible that the greater variety in height caused participants to employ the prototype strategy rather than the double strategy. Further research on this point may reveal a clear justification.

This study has a number of limitations that should be addressed before concluding the discussion, and these are:

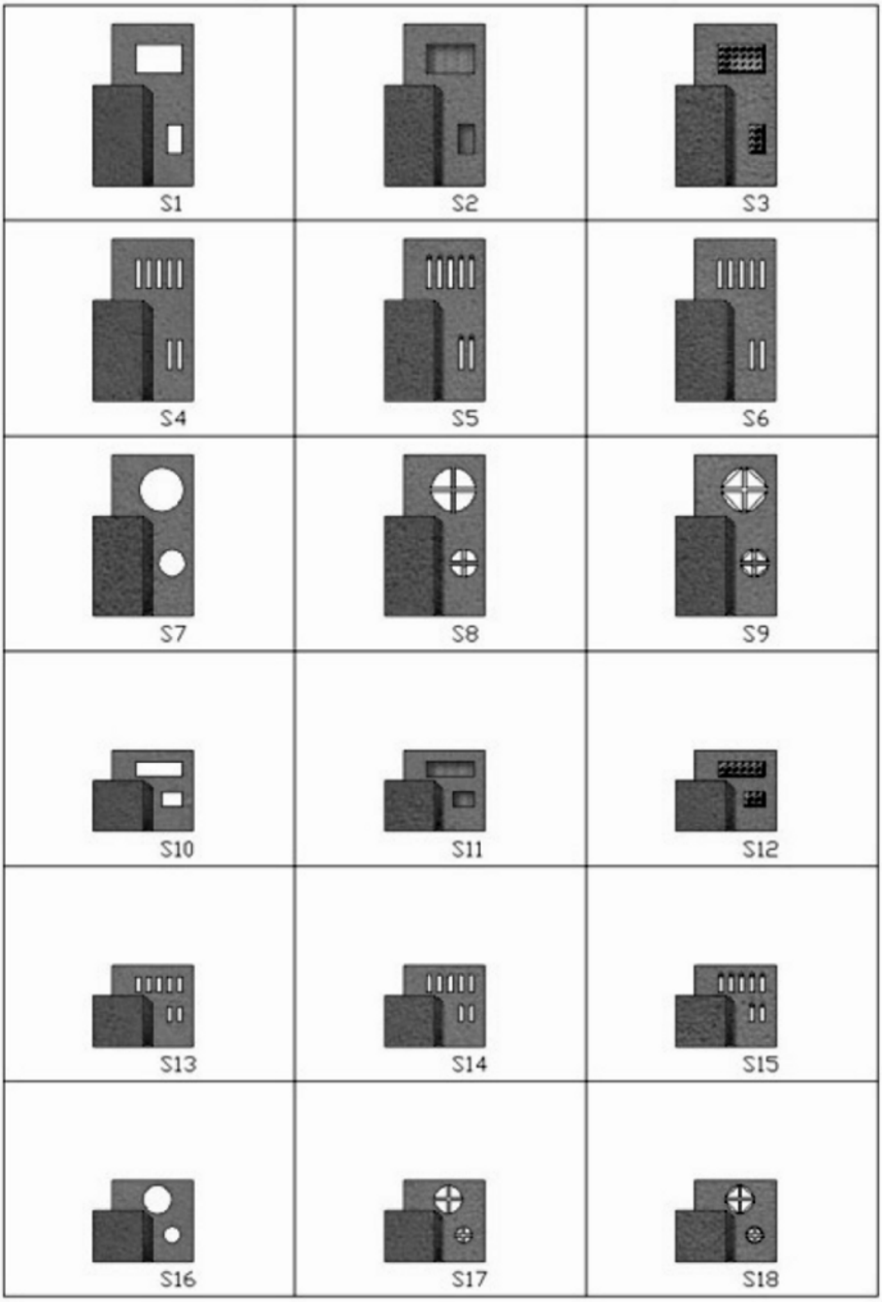
1. The study stimuli were not realistic. They were two-dimensional and included a limited number of design variables;
2. The participants had a relatively short time to learn architectural street façades as architectural contexts (learning groups: A, B, and C). In real life, people require much more time and repeated experiences to establish stable (constant) images of architectural contexts in their real environments.

Accordingly, future research is recommended to address the above limitations. Other, more realistic stimuli with a larger number of variables (including textures, light qualities, colors and architectural details) are recommended for future studies. Also recommended is an investigation of such stimuli as being learned frequently by study participants, to make sure they establish stable images of architectural context. Finally, further research needs to be undertaken on the effect of context composition on which strategy is adopted by perceivers in contextual fit.

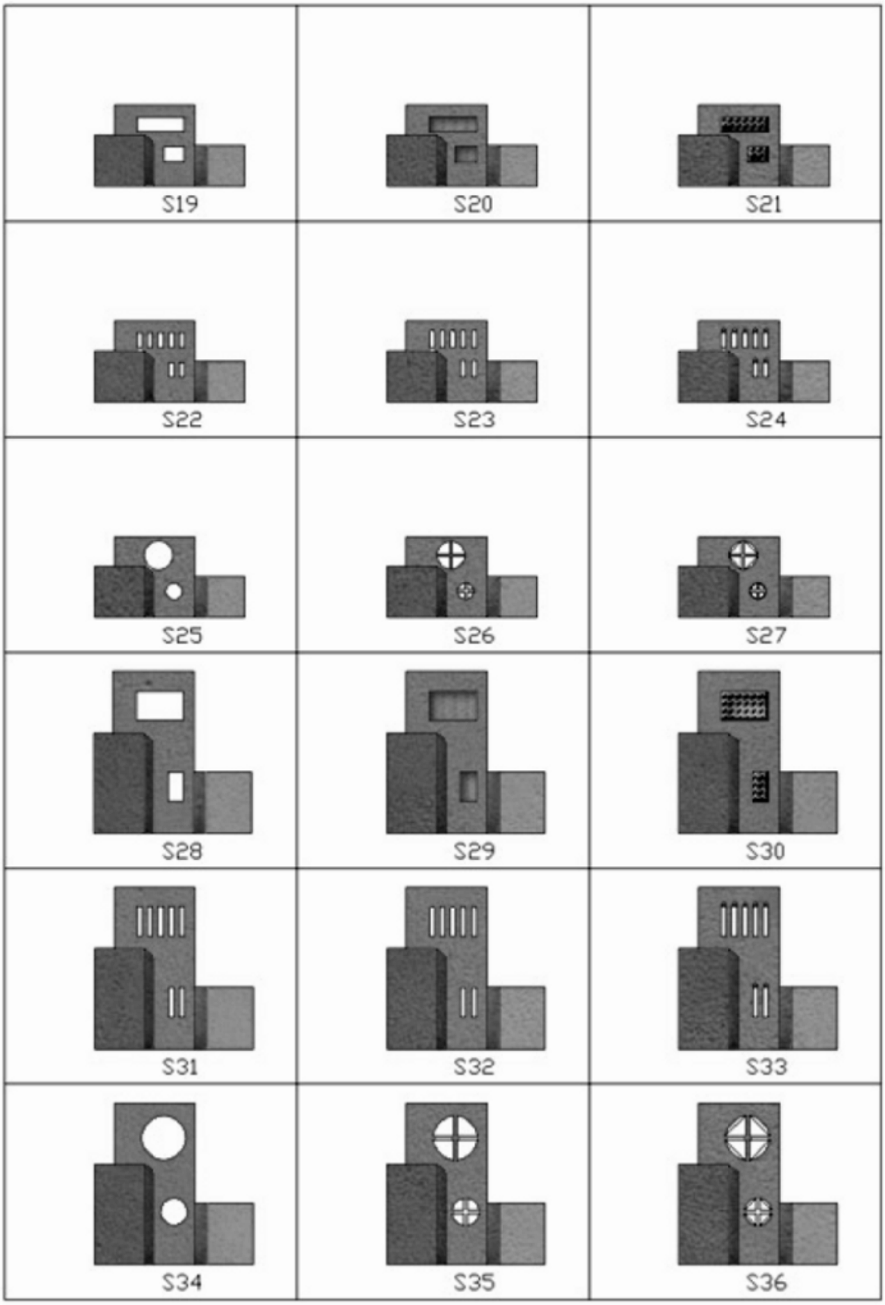
The importance of this research for architectural practice and research comes from the following:

– It is an attempt to further develop scientific and quantitative studies in architectural research;
– It involves people in shaping their visual environment;
– It can be used to develop public policies for aesthetic controls by conducting scientific experiments on public preferences before the regulations are implemented.

Appendix I



Patterns: S1-S18



Patterns: S19-S36

Appendix II

A. The central tendency of a prototype of any chosen category can be determined by measuring the similarity between the prototype and the other categories. Shepard [1962] proposed a method of transforming similarity measures into distance measures, which was modified later by Kruskal [1964]. Shepard proposed that similarity and distance could be related such that the greater the similarity between two stimuli, the smaller the distance between them in a multi-dimensional space. The exact metric can be found empirically for any set of stimuli:

If we have a set of *N* patterns, then, $N(N-1)/2$ similarity measurements data are needed for the scaling algorithm. By taking advantage of the constraints inherent in the data, ordinal scale similarity data can be converted into ratio scale distance measurements. The Shepard-Kruskal distance algorithm follows the Minkowski *r*-metric as follows:

$$d_{ij} = \left\{ \sum_{m=1}^{d} (x_{im} - x_{jm})^r \right\}^{1/r}$$

where:

d_{ij} = distance between stimuli i and j ;
m = subscript for orthogonal axis of the space (m = 1, 2, ... , d);
X_{im} = projection of stimuli i on dimension m;
X_{jm} = projection of stimuli j on dimension m;
r = number of model dimensions which best fit the data.

The two most popular values of r are: $r = 1$, for the city block model, and $r = 2$, for the Euclidean model. The city block model is applicable when the stimuli differ with respect to an obvious and compelling dimension. The Euclidean model is applicable when there is no dominant single dimension, which means that r-value can be 2, 3, or more. Stimulus X_i can be spatially represented within a three-dimensional space by the projections (X_{i1}, X_{i2}, X_{i3}). The distance between the two stimuli X_{i1} and X_{j1} on dimension-1 can be identified as (X_{j1} - X_{i1}). The distance between these two stimuli on dimensions (1 & 2) can be determined according to the following formula: $((X_{j1} - X_{i1})^2 + (X_{j2} - X_{i2})^2)^{1/2}$

B. In our study, the calculations were based on the previously mentioned Shepard-Kruskal distance algorithm, and following Minkowski r-metric as follows:

m = subscript for orthogonal axis of the space (m = 1, 2, 3, 4);
X_{im} = projection of stimuli i on dimension m:

$$X_i = (X_{i1}, X_{i2}, X_{i3}, X_{i4});$$

X_{jm} = projection of stimuli j on dimension m:

$$X_j = (X_{j1}, X_{j2}, X_{j3}, X_{j4});$$

r = 4 (four-dimensional solution).

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Research

Nicola Zabaglia and the School of Practical Mechanics of the Fabbrica of St. Peter's in Rome

Abstract. Nicola Zabaglia, master mason of the Fabbrica of St. Peter's and inventor of many ingenious mechanical devices for restoration, was also the director of the "School of Practical Mechanics" for the education of young labourers. At a time when traditional operational experience was strongly rivalled by the coeval achievements in the theory of mechanics and its effect on building, the work of Zabaglia became an instrument of propaganda. Since empirical practice and the oral transmission of operational knowledge were called into question by the pressing progress of science as well as by new institutions, the works of Zabaglia and his talented students were not only an influential model of cohesion between architecture, building yard and applied mechanics, but also a melancholy epilogue of a practical tradition inexorably condemned to oblivion.

The conflict between empiricism and theoretical science in building

Between the late sixteenth and the early eighteenth centuries the newborn science of mechanics, based on the use of mathematical analysis and on the adoption of novel spatial and structural models, gave rise to a willingness to turn the new devices designed by engineers into working prototypes.

However, in comparison to other fields of application, these achievements will reach the building industry to a minor extent and much later, because of the strong rivalry with a know-how boasting ancient origins, centuries of perfection and unrivalled successes. The inventions by Nicola Zabaglia (1664-1750), master mason of the Fabbrica of St. Peter's in Rome, represent an anomalous and fortunate example of the supremacy of know-how over theoretical knowledge, with significant consequences in practice (fig. 1). The Fabbrica of St. Peter's in Rome was the papal institution in charge for the financial and technical administration of the Basilica's building yard.

At that time, empirical experience in building was gradually giving way to an "analytical rationality", making evident the indissoluble bond – formerly denied by the engineers belonging to the Vitruvian school – between science and technology, and the theoretical approach began to take hold in the antiquated field of building mechanics [Picon 2006]. This achievement occured gradually, and was often based on intuition and long-standing empiricism. Indeed, during the seventeenth century, mechanics, as a branch of physics, had freed itself from its practical origins and initial bond with machines, and the way of thinking of craftsmen, engineers, and even "mechanics" [Dijksterhuis 1971]. Thanks to the contributions by Galileo Galilei (1564-1642) and Isaac Newton (1642-1727), the science of mechanics developed independently of mathematical physics, by investigating the laws of motion and equilibrium, and by banishing the theory of machines to the role of a mere application [Rossi 2000: 191; Mach 2001].

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Fig. 1. Pier Leone Ghezzi, *Portrait of Nicola Zabaglia*. From [*Castelli e ponti* 1743], title page, copperplate engraving; drawing by P. L. Ghezzi, engraving by Girolamo Rossi

Around the turn of the seventeenth century, scientists from the Académie Royale des Sciences in Paris defined the criteria for structural calculation and checking and suggested practical experiments to determine strength values. Thus, a relation between research on statics and building practice was established. Later on, thanks also to the contribution by Bernard Forest de Belidor (1698-1761), strength values were determined on the basis of both mathematical formulae and practical experiments, as well as a synthesis of the two procedures. In the same years Johann Bernoulli (1667-1748) devoted himself to the definition of "dead force" (potential energy) and "live force" (kinetic energy) [Bernoulli 1724] and Joseph-Louis Lagrange (1736-1813) defined the principle of virtual work [Lagrange 1788]. Such topics fascinated in general the large group of scientists who were committed to defining the different branches of mechanics around the turn of the eighteenth century [Capecchi 2002; Dugas 1950], and in particular the Jesuit Vincenzo Riccati (1707-1775), who studied hyperbolic functions and was the author of the treatise *De' principi della meccanica* published in Venice in 1772.

The co-penetration of science and technology, and the new esteem with which technical knowledge was regarded translated into a lively dialogue between mathematics and mechanics. Mechanics correspondingly gained in dignity as a discipline: this was a presupposition for the scientific revolution of the eighteenth century, conceived as a radical and irreversible re-ordering of knowledge.

There existed, however, an evident imbalance between ancient traditions and modern scientific practice, one powerful enough to subvert the very concept of "revolution", as least as regards building practice [Shapin 1998]. This remained long impervious to mechanistic and experimental philosophy, nor was it informed by anything that could be assimilated to a "scientific method", i.e., a coherent, universal and effective sum of cognitive principles and procedures. Though this new theory of mechanics became increasingly known and appreciated, the attempt to mechanise cognitive processes through the formulation of certain prescriptive rules found no place on building sites. Here a consolidated empiricism – heir to the technology of Imperial Rome and of the great building enterprises, and renewed by the experience of the great Renaissance and Baroque architects – still continued to hold sway. This pragmatism rejected the reduction of natural phenomena to mathematical formulae, and was based exclusively on the empirical knowledge gleaned from experience and handed down from the ancients: a body of knowledge perfected over centuries of application and by the gradual improvement of machinery and equipment for construction by technical experts.

Nicola Zabaglia

Among these technical experts was Nicola Zabaglia, who owes his success to the numerous scaffolds built during his long career and to the many restoration works in St. Peter's Basilica. The scaffoldings designed by Zabaglia share a constant care for the conservation of wall surfaces, the safety of workers and the reuse of materials; they represented such remarkable progress in comparison with the usual mechanical building devices that they were used without interruption up to the first half of the twentieth century and abandoned only following the widespread introduction of modern metal scaffoldings.

Zabaglia's devices, emblems of a knowledge based on the intuitive and pragmatic side of the human mind, were designed to permit operations of ordinary and extraordinary maintenance of St. Peter's, whose colossal size hinders even dusting (fig. 2). They were not therefore simple machines, but rather extraordinary temporary scaffolds, designed to support, at the dizzying heights of the basilica, the workers, equipment, materials and machines necessary for restoration works, mosaic and painting decoration, the installation of statues, as well as for placing the iron hoops in the dome.

Zabaglia joined the Fabbrica of St. Peter's, the papal institution in charge of the financial and technical administration of the Basilica's building yard, as a labourer in 1678; in 1691 he figured among the seven labourers working for the architects Mattia de Rossi (1637-1695) and Carlo Fontana (1634-1714). During his career Zabaglia successfully completed several technical undertakings involving the use of scaffolds composed of simple wooden elements that could be controlled by means of a network of ropes driven by hoists operated by men and animals.

Fig. 2. Giovanni Battista Piranesi, St. Peter's southern transept with Fabbrica's workshops and offices [from G. B. Piranesi, *Vedute di Roma* (Roma 1748) pl. 20]

Thanks to his incredible intuition, and with apparent ease, Zabaglia brought to perfection these devices, improving their components, simplifying their counterlath and turning them into movable devices that could be quickly placed where they were needed. Consequently, the Fabbrica of St. Peter's was able to reduce execution costs and times considerably; moreover, downtime was also shortened, as it was no longer necessary to dismantle and reassemble all the components [Marconi 2004].

Thus, timberings, scaffolds and machines became more effective, and even complex interventions could be carried out more quickly and with less material, ensuring the successful completion of the work. Indeed, we owe to Zabaglia the improvement in the technique used to graft and join wooden beams to assemble very large scaffolds and machines.

Zabaglia's career begins in 1695-1696, when he accomplished the transport of the porphyry for Otto II's tomb from the foundry to the Chapel of the Blessed Sacrament. To this end, Zabaglia employed metal levers, hoists and tackles whose dimensions were proportional to the huge weight to be lifted. The success of the enterprise won him the assignment of other important jobs, such as the improvement in 1703 of the pole crane that was used to set up the statues on the sides of the colonnade in the square; the transfer in 1737 of the huge (7.37 m. by 2.24 m.) fresco by Domenichino representing the martyrdom of St. Sebastian to the Mosaic Studio [AFSP: arm. 27, rip. E, vol. 431, p. 22v]; the construction in 1735 of the magnificent scaffold, "touching neither the floor nor the walls", used to restore the friezes in the vault of the tribune in the church of San Paolo dentro le mura; as well as several other transports of statues, huge tombstones and dedicatory monoliths. However, his undisputed masterpieces were the scaffold to restore the friezes in the great dome of St. Peter's and the one built for the restoration of the stucco works, made by Giuseppe Lironi between 1731 and 1734, in the nave. In the same year Zabaglia supervised the transport of the Sallustian obelisk (62.25 spans, that is, about 14 m. high) from Villa Ludovisi to the piazza of S. Giovanni in Laterano. According to the (unsatisfied) wish of Pope Clemente XII Corsini (1730-1740), the obelisk was to have stood in front of St. Peter's Basilica [D'Onofrio 1992: 355-368; Felici 1952].

The technology of scaffolds

The scaffolds invented by Zabaglia consisted of two broad groups: those designed to assist in their work painters, stucco and mosaic decorators, as well as masons, carpenters and stone-cutters; and the ones aimed to support more complex jobs, carried out within the Vatican as well as outside (thus the importance of the Fabbrica of St. Peter's). Scaffolds can be divided in ground and suspension framework.

Ground scaffolds consist in a main structure made of *ritti* or *candele* (vertical poles); these are about 16 cm wide, usually double and joined by iron stirrups and ropes, placed 2 m. apart and leaned against two opposite walls. Stringers are linked to the *ritti*, so to define several tiers of trestles and leave a free space between the flooring and the ceiling as high as a man, i.e., about 1.90 m. On the stringers are laid the *travicelli*, which in turn support the planking. Ground scaffolds, based on principles of simplicity, reversibility and strength, should be solid enough to support the stress produced by lifting materials; therefore, they are anchored to short, strong (chestnut) beams embedded right in the wall structure and blocked in their slots by wooden wedges. The multistorey scaffold built by Zabaglia in 1735 belongs to this type, though improved in some of its components; it was used in the tribune of the church of San Paolo fuori le mura for the restoration of the vault [Zabaglia 1743: pl. XX]. It stood out for Zabaglia's mastery in reaching the height of the tribune up to the cornice "by means of large trestles", surmounted by several tiers up to the top of the vault [AFSP, arm. 12, rip. D, vol. 4b, fasc. 29, p. 1030] (fig. 3).

Among the so-called suspension scaffolds was the big planking erected by Zabaglia for the restoration of the mosaic friezes in the main dome, acrobatically constructed on the cornice up to the point allowed by the curvature of the dome. Such complex, though non-invasive, scaffolding was designed with the utmost respect for the structure of the dome: it was hooked to two *ritti* that come down from the lantern and to which the large ledgers are fastened forming a *bilancia* (a light hanging scaffold) "convenient to work at that height" [AFSP: arm. 12, rip. D, vol. 4b, fasc. 29, p. 1031v]. At the opposite end, the *ritti* are secured to strong iron hooks. Here again, safety, respect for the structure, simplicity and reversible assembly guide Zabaglia's inventions; having, as a real engineer, "once established some chief ideas, [he] can apply them in any given case" [AFSP: arm. 12, rip. D, vol. 4b, fasc. 29, p. 1031v].

Thanks to the crucial improvements made by Zabaglia, flexible use and adaptability to various contexts became the main features of scaffolds and timberings, which are characterised by versatility, convenience, safety and economy. In order to facilitate his precious talent as a "mechanic", Zabaglia was relieved of the daily tasks assigned to the other workers in the Fabbrica and was granted the use of a studio where he could test the mechanical prototypes he invented by making working scale models [AFSP: arm. 12, rip. D, vol. 4b, fasc. 29, p. 974v]. His inventions thus made it possible for the empiricism of building technology to compete with the advances in theoretical research, and thus were accorded official sanctioning by publication.

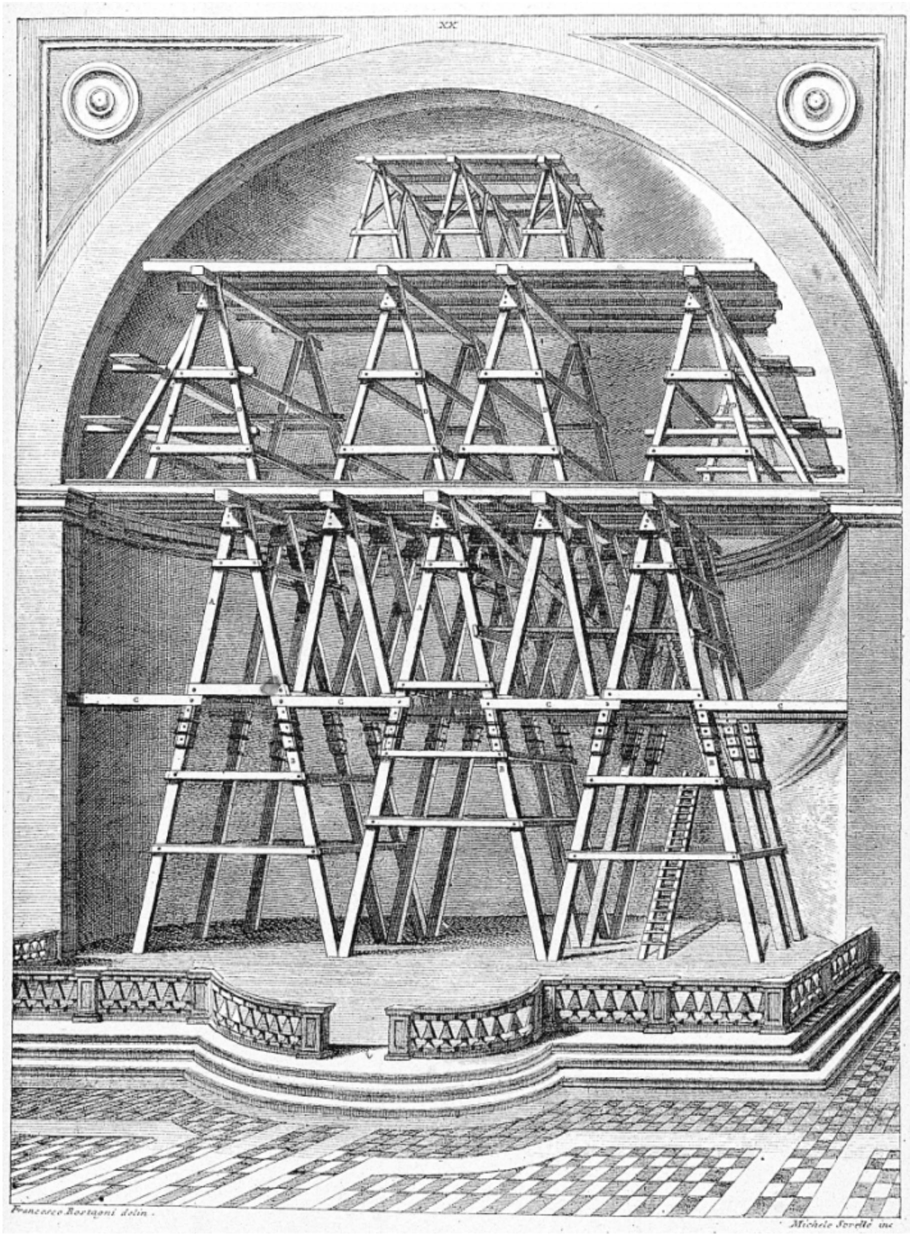

Fig. 3. Scaffold for the tribune vault of S. Paolo fuori de mura. From [*Castelli e ponti* 1743: pl. XX], copperplate engraving; drawing by Francesco Rostagni, engraving by Michele Sorello]

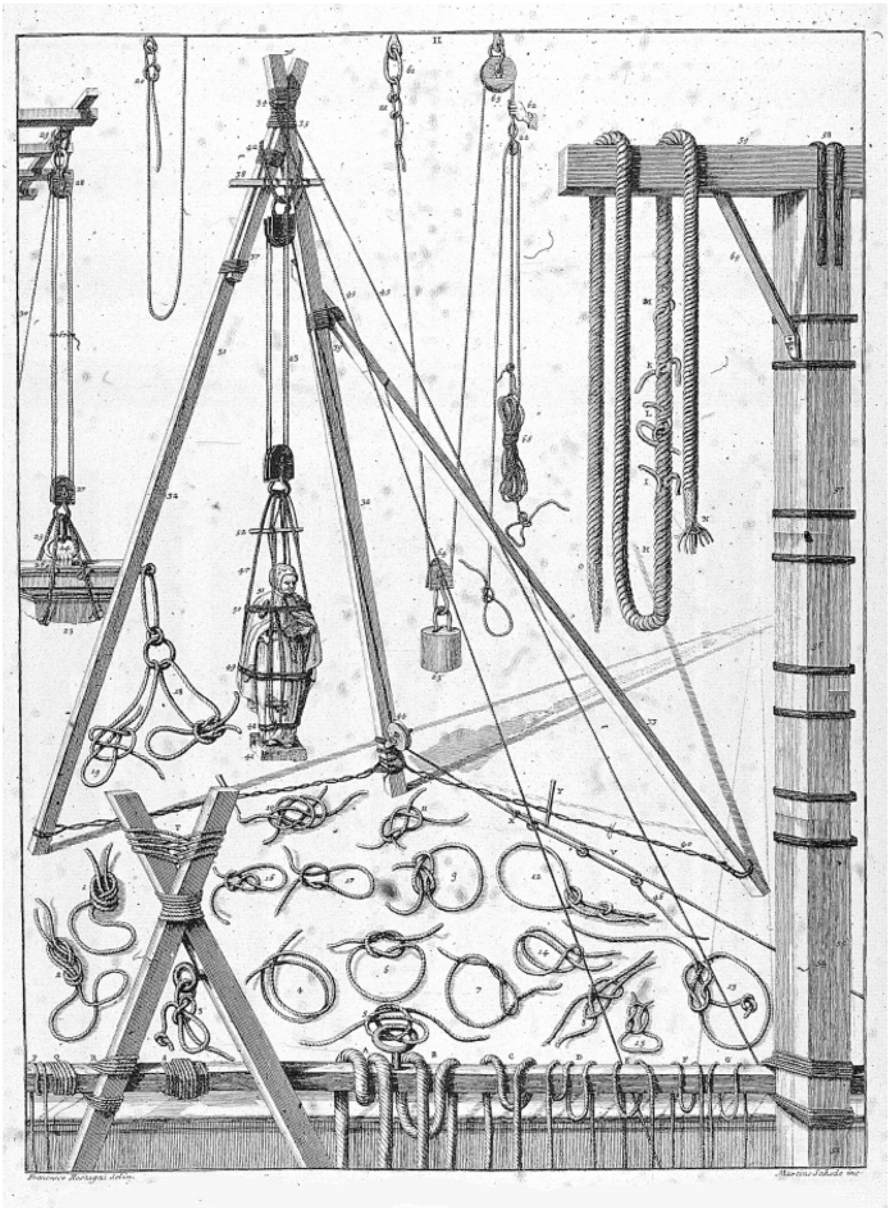

Fig. 4. Hemp ropes, simple hoisting machines and lifting blocks – a combination of pulleys and ropes which allows heavy weights to be lifted with least effort. From [*Castelli e ponti* 1743: pl. II], copperplate engraving; drawing by Francesco Rostagni, engraving by Martino Schede

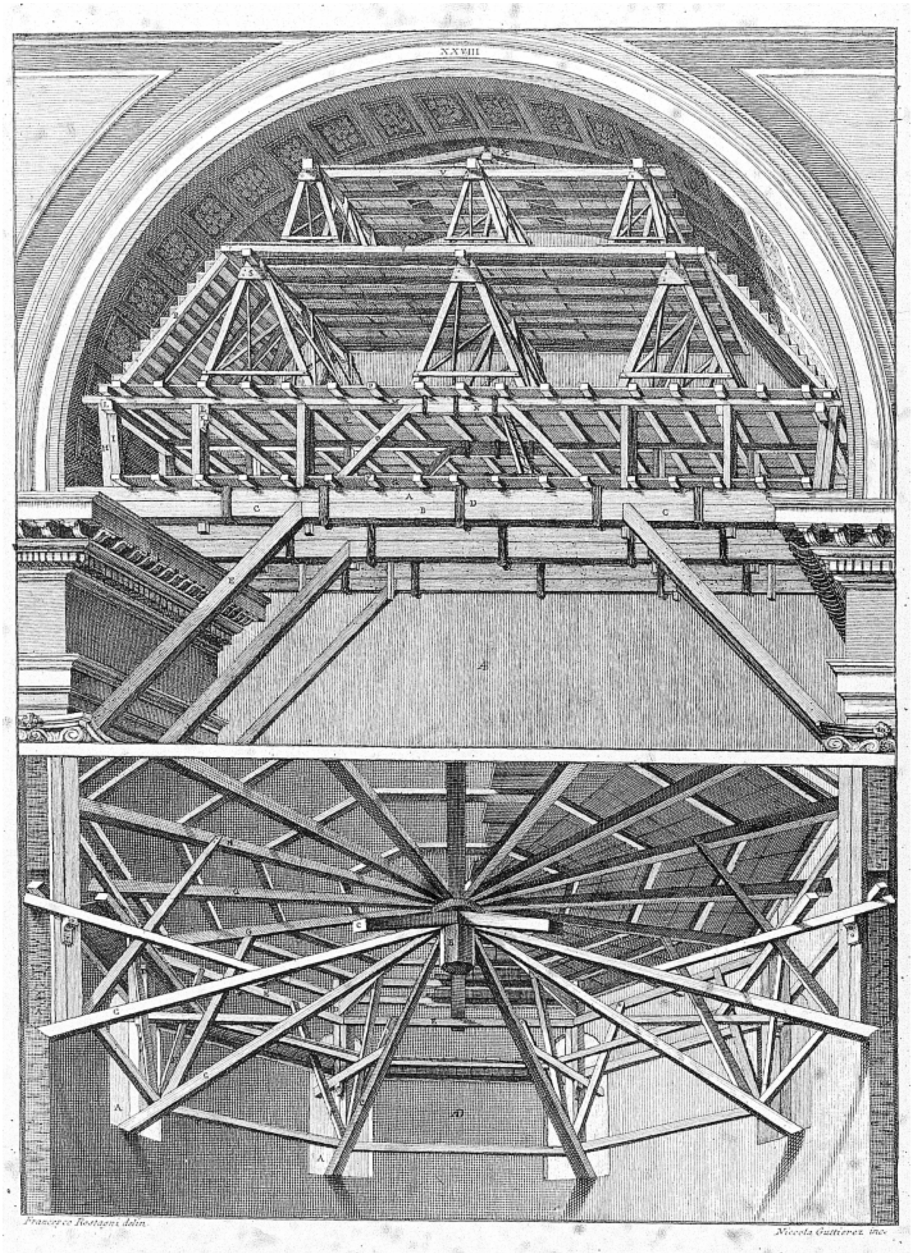

Fig. 5. Scaffold for octagonal vault of St. Peter's Sacristy. From [*Castelli e ponti* 1743: pl. XXVIII], copperplate engraving; drawing by Francesco Rostagni, engraving by Nicola Gutierrez

This occurred in 1743, when a book entitled *Castelli e Ponti di Maestro Nicola Zabaglia* was published under the patronage of high Vatican officials, including Ludovico Sergardi (1660-1726) and Lelio Cosatti (1677-1748), who wrote the figure captions [Marconi 2008a]; they intended to show to the entire world the talent of the technicians in the Fabbrica of St. Peter's and the leading role of the Fabbrica in Rome's building industry. The book contains fifty-five plates depicting Zabaglia's inventions, as well as tools and devices for everyday use. It represents a genuine compendium of know-how, validating the old pragmatism in the face of the new, pressing scientific progress. It aimed to perpetuate the operational tradition perfected over three centuries of activity in the building yards of the Vatican, already made famous by Domenico Fontana's *Della Trasportatione dell'Obelisco Vaticano* (1590), and by Carlo Fontana's *Templum Vaticanum et ipsus origo* (1694). However, *Castelli e Ponti* reappraised the role of the "mechanics" in building, who, in contrast, had been harshly criticised by Fontana, who vigorously claimed the supremacy of the architects' theoretical experience (see [Fontana 1694: Bk. II, ch. XII, 101]; see also [Hager 2003: XXXIV-XXXVII]) (fig. 4).

These publishing initiatives show the intrinsic conflict between practical experience and theoretical knowledge that existed in the building industry in the eighteenth century, and strengthened Clemente XI's mission to provide the Catholic world with tangible evidence of the Church's building activity and of the considerable intellectual, technical and economical resources lavished for the completion of St. Peter's, in defence of Christian doctrine.

The rich illustrations in *Castelli e Ponti* reproduced structures, components and material consistency of building scaffolds, machines and tools with photographic detail. The text was divided into three main sections and shows tools, lifting machines and scaffolds already in use in the basilica and in everyday building practice, together with a fascinating assortment of the scaffolds invented by Zabaglia for the maintenance works in St. Peter's (fig. 5).

Critical approval of Zabaglia's work was immediate and long-lasting. As years went by, the interest of architects, engineers and technicians in Zabaglia's work grew stronger, to the point that it came to represent the only and indisputable reference point for designing similar devices [AFSP: arm. 28, rip. C, vol. 491, p. 7, n. 25; Debenedetti 2006: 91].

Such were the success and the didactic value of *Castelli e Ponti* that the book soon sold out: on 22 July 1817, the Sacred Congregation of St. Peter decided to reprint it. The reprint was enriched with six new plates depicting the scaffolds made by Zabaglia's most skilful pupils, including Pietro Albertini and Angelo Paraccini [AFSP: arm. 12, rip. D, vol. 4b, fasc. 29, pp. 973r-974r].

The School for Practical Mechanics

The *Memoirs* on Zabaglia's life enclosed with the second edition of *Castelli e Ponti* mention a so-called School of Practical Mechanics, founded, perhaps about 1719, to "pass Zabaglia's rules, methods and mechanisms on to other labourers" [Zabaglia 1824: vi]. To date, however, no reliable evidence of the actual foundation of this school has emerged from the records in the archives of the Fabbrica of St. Peter's. Therefore, most likely the "school" was rather a small group of labourers collaborating with Zabaglia in the most demanding jobs and forming with him a "task force" appointed to solve various technical and mechanical problems [AFSP: arm. 12, rip. D, vol. 4b, fasc. 29, p. 966v]. Thus was created the first group of workmen in St. Peter's, now called "sanpietrini": these were young people already expert in the carpenter's, the mason's, the stone-cutter's, the smith's, the stucco decorator's trades who "would be ready and suitable for any job at any time, with no need to get mixed up with architects, engineers and outsiders, who never earn enough and therefore tend to delay the work" [Grimaldi 1998].

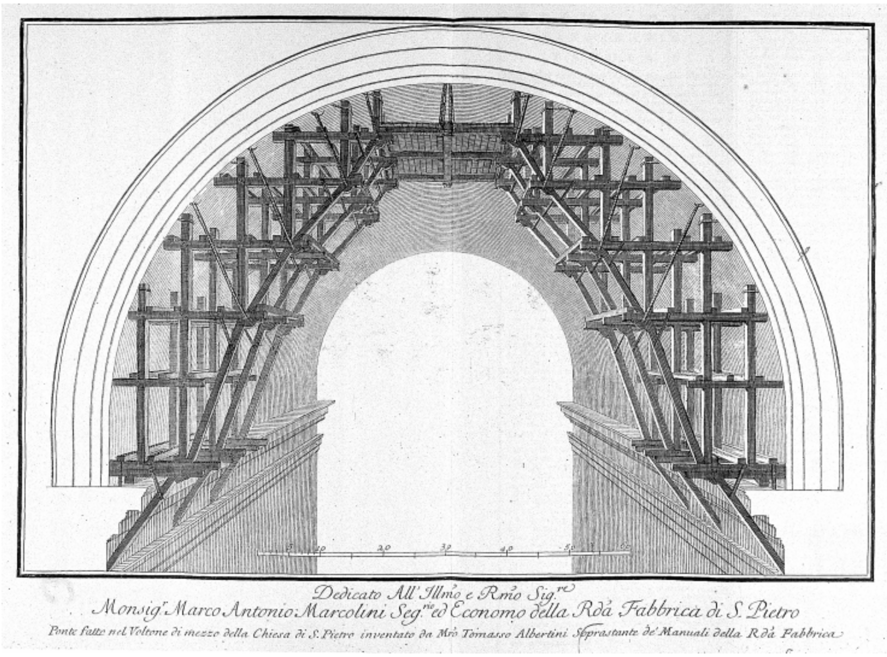

Fig. 6. Scaffold for restoration work at great barrel vault of St. Peter's designed by Tommaso Albertini. From [*Castelli e ponti* 1743: pl. LV], copperplate engraving

The experience acquired working side by side with Zabaglia moulded his apprentices better than any school, turning them into extremely skilful technicians. This would account for the rumour about a "school of mechanics" in St. Peter's; this hypothesis is corroborated by records reporting the names of a small group of assistants, always the same ones, who collaborate with Zabaglia throughout his long career. These include his brother and his nephew (Alessandro and Antonio Zabaglia), but also Angelo Paraccini, Pietro Antonio and Giovanni Corsini, and Tommaso Albertini, who, after Zabaglia's death in 1750, succeeded him as master mason and would later even be promoted to the position of supervisor of the Fabbrica of St. Peter's, that is, the most prestigious position except for the architect, which he held from 1773 to 1787 (fig. 6). After him, his son Pietro (1788-1793) and Angelo Paraccini (1794-1800), another skilful pupil of Zabaglia's, became supervisors [AFSP: arm. 51, rip. G, vol. 81]. Paraccini continued to train young apprentices, choosing among them "the fittest to go in the air and practise mechanics for the tasks necessary for the maintenance of the great church of St. Peter" [AFSP: arm. 52, rip. A, vol. 85, p. 336r].

Nevertheless, "not everyone is Zabaglia, who was a born mechanic", and competition grew tougher and tougher [AFSP: arm. 51, rip. G, vol. 81, p. 439]. Though later than in other Italian and European contexts, first and foremost France, from the end of the seventeenth century the inexorable transformation of the technical profession also occured in the Papal States. In the building industry, this translated into a growing distinction between the intellectual role, peculiar to the designer, and the operational role. In the profession of architecture, practice slowly started giving way to theoretical training, boosted by the advances in mathematics, physics and mechanics. If, in theory, experience in the yard had to co-exist with science and historical-artistic knowledge and play a role in the training of the architect-engineer, in practice, solving of the most complex operational problems was assigned to expert master masons or supervisors, above all, those working in the Fabbrica of St. Peter's [Di Marco 2006; Picon 2002].

The debate between theory and practice

The exquisitely Roman debate within the Accademia di San Luca, widely studied by historians, was emblematic of this conflict [Cipriani & Consoli 2007; Giusto 2003; Cerutti Fusco 2000]. The statute of the Academy was significantly modified during the first decade of the nineteenth century.

Less known, if at all, is the contribution of the Fabbrica of St. Peter's to the debate on the theory and practice of the building profession. Towards the end of the eighteenth century, thanks to the *motu proprio* of Pope Pio VI Braschi (1775-1799), on 14 February 1794 the Papal School of Design Principles was founded; it was the fourth among the Christian Schools founded in 1789 in San Salvatore in Lauro. The school, first directed by architect Andrea De Dominicis (deceased in 1807) [Debenedetti 2006: 332-335], provided training for "artists, mechanics and young engineers appointed to carry out the maintenance works of the Vatican temple" [AFSP: arm. 45, rip. A, vol. 55, n. 212]. This school represented an educational alternative to the prestigious theoretical training provided by the Accademia di San Luca, a sort of workshop intended to train, rather than artists, *artieri*, i.e., experts in mechanics capable of ensuring proper execution of the work and satisfaction of the clients. If on one hand the necessary condition for admittance to the School of Design was indeed *not* to belonging to an architects' studio, on the other hand the structure and the organisation of the school were closely modelled on those of the Accademia di San Luca; it even drew on the tradition of the "concorso", that is, the final test [Marconi, Cipriani, Valeriani 1974].

In 1814, after the restoration of the Papal States following the Napoleonic period, the School of Design Principles was transformed into the Public Studio of Civil Architecture, but was still open to the youth who were redundant in the Fabbrica of St. Peter's corpus of workers; they were obliged to attend the school twice per week, which became the fundamental requisite for aspiring labourers [Di Sante 2008].

Thanks to the foundation first of the Christian Schools, and then of the Papal Studio of Arts, the Fabbrica of St. Peter's was confirmed as the interpreter and upholder of operational knowledge. The Regulations of the Papal School of Arts, published in 1822 but preceded by a first draft in October 1821, made clear the need for training new generations of *artieri* to keep alive the precious fund of technical and mechanical knowledge amassed by Rome's building industry throughout centuries of fruitful activity (see [AFSP, arm. 12, rip. F. vol. 10, fasc. 18, *Regolamenti per lo studio pontificio delle arti stabilito nella casa religiosa delle scuole cristiane presso San Salvatore in Lauro*, Roma, 1822]).

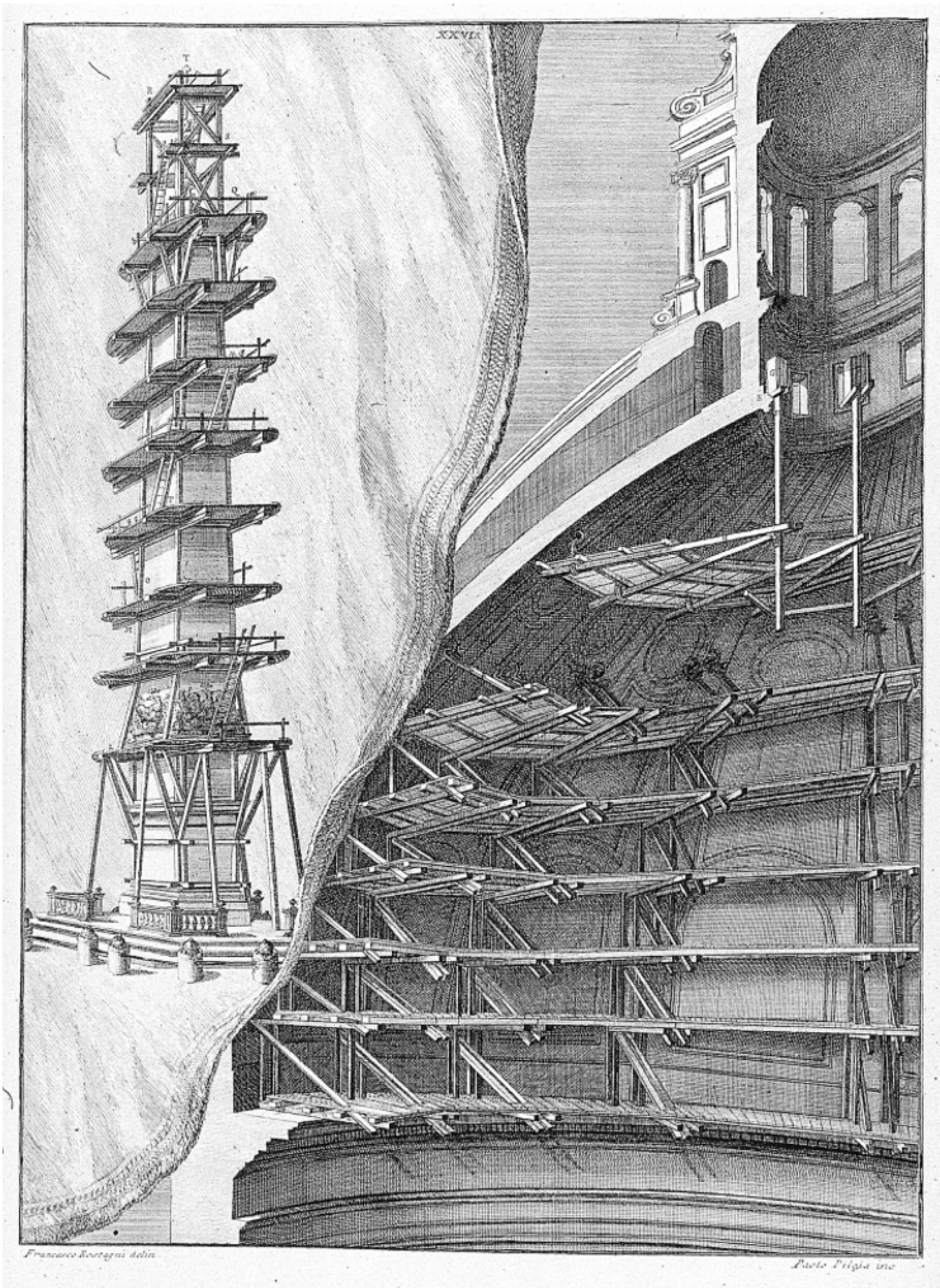

Fig. 7. Scaffolds for restoration work on the Vatican obelik and the dome of St. Peter's. From [*Castelli e ponti* 1743: pl. XXVI], copperplate engraving; drawing by Francesco Rostagni, engraving by Paolo Pilaja

On the opposite front, ruled by the necessity of science, the regulations of the School of Engineers, founded following the *motu proprio* of Pope Pio VII Chiaramonti (1800-1823) on 23 October 1817, aimed at training civil engineers "methodically and fully ... in any knowledge needed for practising the art". The school intended to promote mechanical science by means of direct observation and practical experience, "to ensure the application of principles and theories to any practical case" [Biral & Morachiello 1985; Giuntini & Minesso 1999]. Theory and practice were thus in competition again; however, if scientific progress had already identified specific distribution channels and fields of application, the building industry was clearly taking pains – as exemplified by the urgency felt even by the Fabbrica of St. Peter's, the historic keeper of a long-time praxis – to train its scaffold builders, masons, carpenters and mechanics first in the theoretical principles of mechanics and then in the actual construction of timberings, scaffoldings, tie bars and underpinnings [AFSP: arm. 63, rip. C, vol. 37, f. 5, n. 21].

A few years later, in 1822, to define the general didactic guidelines, the regulations of the studio explicitly mentioned *Castelli e Ponti di Maestro Niccola Zabaglia,* referred to as a fundamental instructive tool to train skilful mechanics, capable of keeping the "art of scaffolding" alive. The book met the specific teaching needs of the course in Practical Mechanics, where architects and engineers were taught the "theoretical rules of this science that apply to the practice of scaffolding, and the cheapest and most effective machines both to balance and to move weights" [AFSP: arm. 63, rip. C, vol. 37, f. 5, n. 21]; to this end Zabaglia's work was repeatedly referred to as the expression of the highest technical standard reached by the operational empiricism of Rome's building industry (fig. 7).

Towards the second half of the nineteenth century, the need to train craftsmen for the building industry and at the same time to teach a trade to underprivileged youth was stated in the statute of the School of Arts and Trades and of the Professional School of Technical Arts of St. Michael's Hospice in Rome, published by Giacomo Lovatelli in 1875 [Lovatelli 1875; Fea 1834: II, 600; Toscano 1996]. Again, there was a section of mechanical arts, featuring a six-year teaching module with compulsory attendance at theoretical lectures on subjects such as applied geometry, drawing of models and design of machines, and practical applications in the workshop [Lovatelli 1875: 61-62]. The school conformed to the teaching schemes of other state technical schools, but also added elements of physical mechanics, mechanics and kinematics. Thus, it was modelled upon the French schools of arts and trades "that are certainly the best ones in Europe, so that a comparison can be made" [Lovatelli 1875: 69].

Nevertheless, Zabaglia's immediate, practical lessons survived that revolution and arrived the twentieth century with their authoritativeness undiminished. The new edition of *Castelli e Ponti* in 1824 was as successful as the previous one, despite the proliferation of compendia of mechanics and handbooks of engineering that were being published at the same time [Guenzi 1993]. The seventeenth- and eighteenth-century technological tradition was drawn on by the devices shown in the nearly contemporary *Architettura pratica* by Giuseppe Valadier (1828-1839), in the famous *Encyclopédie* by Denis Diderot and Jean-Baptiste Le Rond d'Alembert, published in Paris 1751-1780, and in the compendium by Girolamo Masi entitled *Teoria e pratica di Architettura Civile per istruzione della gioventù specialmente romana*, published in Rome in 1788, which contained an interesting series of copperplates (some of which were directly derived from *Castelli e Ponti*) devoted to the building techniques and tools that could be used to introduce young people to building practices (fig. 8).

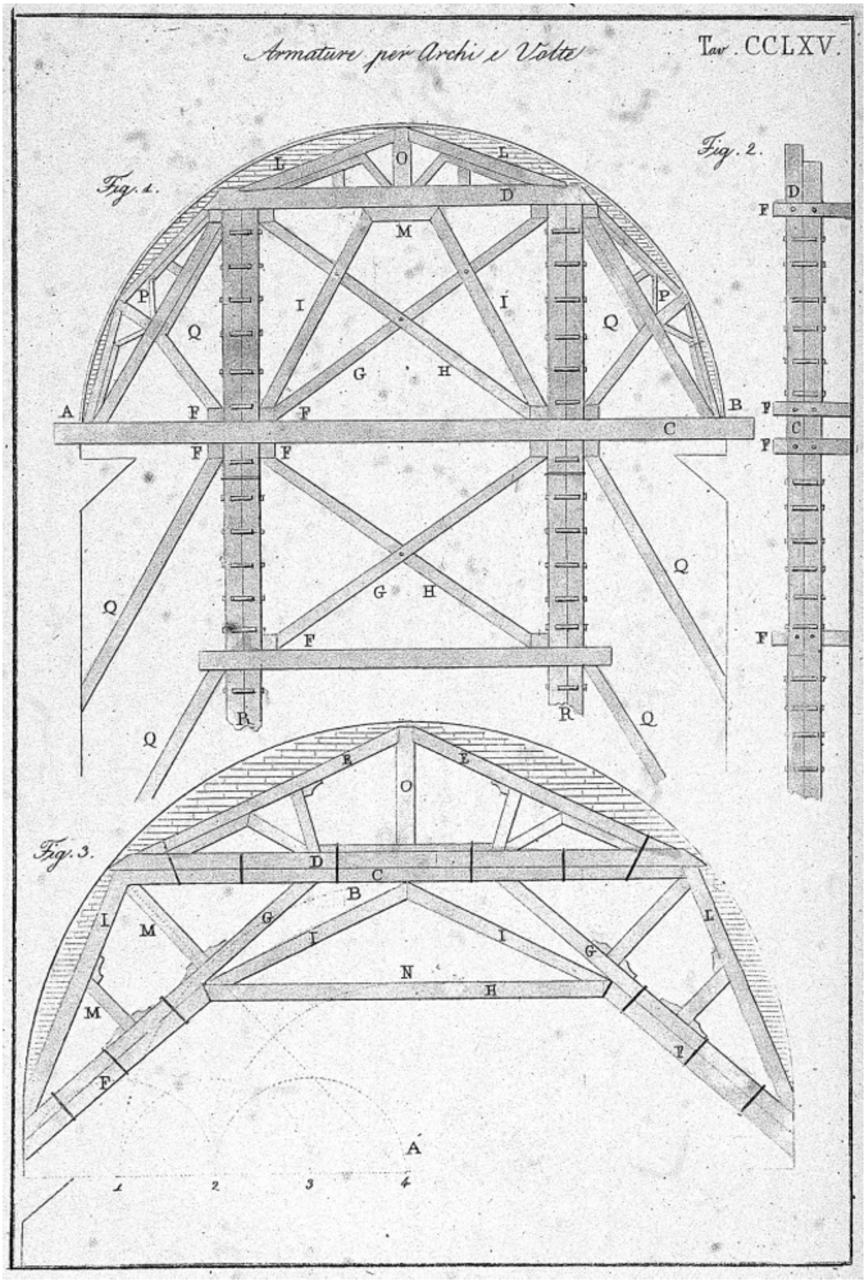

Fig. 8. Arches and vaults timberings [from G. Valadier, *L'architettura pratica* (Roma 1833) IV, pl. CCLXV]

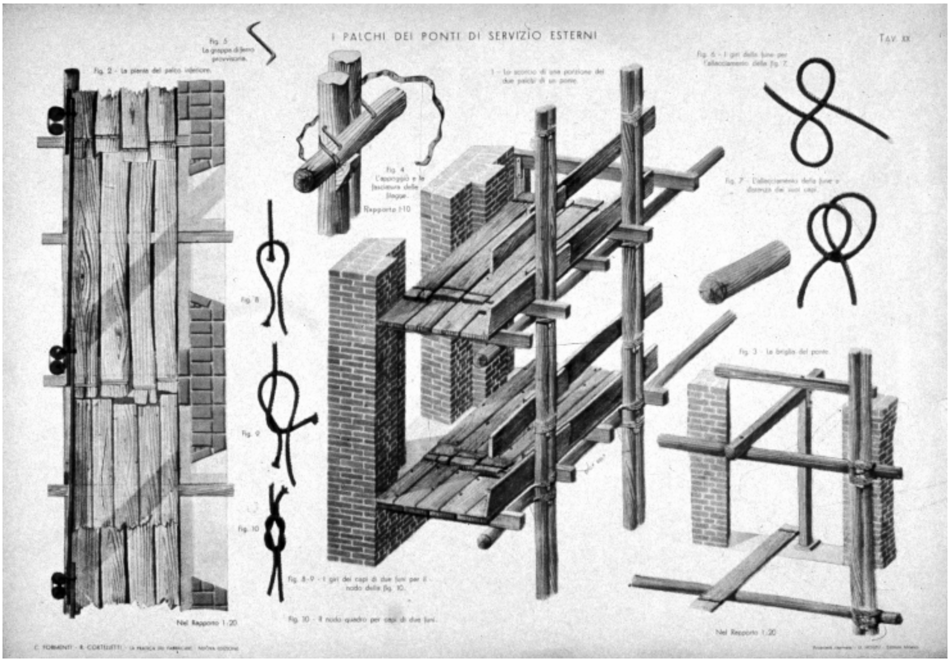

Fig. 9. Building exterior scaffolds with constituens and bindings [from C. Formenti, *La pratica del fabbricare* (Milano 1893) I, pl. II]

Meanwhile, work in St. Peter's proceeded. At the end of the eighteenth century, a new wing was added to the old Fabbrica: this was the Sacristry, which housed the new Mosaic Studio. Machines and scaffolds used by the Mosaic Studio were kept in the rooms on the upper floor, and the old Mosaic Studio was turned into an exhibition room for the models of the basilica and its parts, as well as of machines and scaffolds designed by skilful architects and engineers for its construction or restoration, models that were extremely interesting from the architectural and mechanical points of view, "to amaze laymen and educate professors" [AFSP: arm. 15, rip. F, vol. 151].

Thus is evident the pertinence of the devices designed by Zabaglia and his pupils, whose models represent an extraordinary technological repertoire [AFSP: arm. 12, rip. D, vol. 3, pp. 43-47; Hager 1997: 168; Cascioli 1924; Chattard 1767: III, 137]. In their structure and functionality, these models were identical to the actual prototypes, and reaffirmed the validity of traditional building practice. Therefore it is not surprising that in 1886 Andrea Busiri Vici praised the skilfulness of the then supervisor of the Fabbrica of St. Peter's, Guglielmo Guglielmetti, who designed "the scaffolds for the internal dome, the tambour and the pinnacles, and those for the new external coverings, and the scaffold for raising the Council's column" [Busiri Vici 1886: 13]. Guglielmetti's work was perfectly in line with that carried out by the great technicians who preceded him, from Fontana to Zabaglia and Tommaso and Pietro Albertini. At the end of the century, another of the Fabbrica's supervisors, Ercole Scarpellini, built the huge (26 m. wide) arched scaffold, composed of several tiers of roof trusses, raised on 16 November 1897 to the top of the main vault in the nave, above the statue of St. Peter ([AFSP: arm. 12, rip. D, vol. 4a, pp. 934-937]; see also

La voce della verità, XXVII -XXVIII, November 1897–October 1898). The colossal scaffold, needed to restore the gilding on the vault, was called by contemporaries "a daring and well-designed work, indeed up to the artistic traditions of the first temple of Christianity". The scaffold, which weighed about nine tons, was laid upon the big cornice of the vault, at a height of 30 m., and consisted of four tiers, whose components were fastened by iron stirrups and graft joints, which allowed the artists to work comfortably and safely [Marconi 2008 a-b]. To meet the various requirements of the work, the big "arched scaffold" could slide along the cornice thanks to pulleys and wooden sledges smeared with soap, exactly as in Zabaglia's devices (fig. 9).

Conclusion

Zabaglia's works demonstrated the reliability of traditional operational practice and the effectiveness of the so-called "Roman construction system", and were carried up to the beginning of the twentieth century, thanks to the work of his pupils and heirs.

On the basis of the models of Zabaglia's devices, left "for the benefit of the Reverenda Fabbrica" [Fortini 1896: 195], new building mechanical devices were designed, and gradually improved by using new materials and types of propulsion.

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Research

Mathematics for/from Society: The Role of the Module in Modernizing Japanese Architectural Production

Abstract. This paper presents an examination of the process of the development of module in the works and theories of Japanese architect Ikebe Kiyoshi (1920-79). Ikebe based his idea of module on the belief that "Beauty is Mathematics." He applied his ideas of module in various ways from the 1940s to the 1970s. Analyzing his ideas and works against their historical background, the social and creative meanings of the idea of module and of mathematics in architecture will be re-examined. This allows us to see how Ikebe developed his ideas of module from a characteristic mathematical approach, and how he developed his idea of mathematical logic into his creative theories based on the flexible nature of people's lifestyles and social conditions. Going beyond the cultural and social differences and the limitations of Le Corbusier's Modulor, the idea of module as the method for organizing human space in a harmonious manner was reframed in Ikebe's works, and was developed in a more flexible mathematical way.

Introduction

In 1946 Le Corbusier created a drawing of his idea of Modulor with the aim of enabling effective industrial mass production of architecture by unifying three factors in the architectural module: human scale; the visual harmony of the Golden section; and the imperial/metric measurements based on the Fibonacci series. Nevertheless, in a 1997 survey of Japanese architects by the periodical *Kenchiku Gijutu*, most responded negatively to the idea of the module, and many recognized it as a fixed numerical system based on abstract mathematics. (To be more precise, the responses were "it disturbs the freedom of design by equalization of measurement"; "it equalizes the quality of space", and "it disturbs the variety of designs".) On the other hand, the idea of standardization of measurements has been consistently important in the creation of architecture and public spaces, and particularly in mass social housing in global society.[1] However, architects hardly make any significant contributions towards resolving these difficult issues because their profession has become oriented toward formal design. The narrowness of architects' understanding of the idea of module and their inability to contribute to the discourse on social housing are concomitant, and signify the divisions within architectural design, production systems, the reality of human life, and the representation of social ethos. By referring to the Greek notion of modulus, Heidegger [1971] explained that selection of measurement is a human attempt to create a society based on benevolence. In that sense, the idea of module implies the symbolic role of mathematics in creating social ethos in architecture. In order to resituate the architectural profession within the social context, it is important to re-examine the idea of module in a wider perspective.

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In this paper, I will examine the process of the development of module in the works and theories of Japanese architect Ikebe Kiyoshi (1920-79), who examined the idea of module based on his belief that "Beauty is Mathematics." Ikebe applied his ideas of module in various ways from the 1940s to the 1970s. Through an analysis of his ideas and works against their historical background, the social and creative meanings of the idea of module and of mathematics in architecture will be re-examined. This study clarifies how Ikebe developed his ideas of module from a characteristic mathematical approach, and how he developed his idea of mathematical logic into his creative theories based on the flexible nature of people's lifestyles and social conditions. Going beyond the cultural and social differences and the limitations of Le Corbusier's Modulor, the idea of module as the method to organize human space in a harmonious manner was reframed in Ikebe's works, and was developed in a more flexible mathematical way, influencing future architecture.

The Modulor of Le Corbusier

For the purpose of this study, I would like to briefly summarize Le Corbusier's explanation about the factors and the process of organizing his idea of Modulor as described in his book *Le Modulor* [1954]. Le Corbusier started working on the validity of the Golden section in designing architecture and discussed the ideas of standardization for an architectural unit and its system of mass production in *L' Espri Nouveau* from the beginning of the 1920s. With a group ASCORAL (Assemblee de Constructeurs pour une Renovation Architecturale) in 1942, which opposed the average value measurement system between imperial and metric of the Association Francaise de Normalisation (AFNOR), he began working on the unification of the proportion grid, the Golden section and human scale. After World War II, they put absolute size into the geometrical analysis of the proportion grid, and discovered that the Fibonacci series could organize the measurements derived in his system. Subsequently Le Corbusier started to advocate his idea of Modulor worldwide as a set of measurements to unify the imperial and metric measurements for the industrialization of architectural production, thus applying it in his projects, such as the Unite d'habitation in Marseille from 1945 to 1947.

Here, I would like to point out two issues. First, in the concept of Modulor, Le Corbusier's idea of mathematics was mainly geometrical. The Fibonacci series was appropriated into his system after his application of absolute size into the system. Second, his idea of Modulor primarily aimed at realizing the unification of imperial and metric measurements, not by a convenient averaging method, but by his universal ideology for architecture.

The background of the creation of a national module in Japan

Shaku, a 30.303-cm. module system,[2] was originally introduced into Japan from China, and was developed in the apprenticeship system based on the dominant role of carpenters and on their canonical rules. The *shaku* module was further elaborated in the Edo period by master carpenters for the organization of a coherent structural system called *Kiwari* (sizing of timbers).[3] Later, the columniation systems of *Kiwari* were developed further in *Hashirawari* (arrangement of columns).[4] Particularly in the *Sukiya* style, the placement of the Tatami mat in the whole area of the room became common, and the sizes and arrangement of the Tatami mat became systematized in *Tatamiwari*. In this process, the size of the Tatami mat became unified and linked to the measurement of the Japanese body. Thus, the structural and material balances of Japanese architecture were closely united with its spatial proportions, functions, ornamentations, detailed arrangement of facilities, and human scale.

However, the *shaku* module differed according to geographical area. Examples are *Kyoma* in the western part of Japan, which is based on the distance between the surfaces of column and has a 6.3 *shaku* module, and *Inakama* in the east, which is based on the distance between the centers of columns, and has a 6 *shaku* module. The sizes of the Tatami mats differ accordingly. The cutting of timbers and the application of *Kiwari*, *Hashirawari* and *Tatamiwari* were all controlled by the carpenters, whose knowledge tended to be based on their local and conventional resources and networks, which were different from mechanical standardization.

Discussions to establish a national module

After the Meiji reformation, the Japanese government built many institutional buildings in Western styles, and the metric system became symbolic not only of creating fire resistant cities but also of rationalizing and modernizing architectures, and of people's lifestyle. In order to industrialize the nation, the government continuously tried to replace the *shaku* module with the metric system and to modernize the traditional carpentry system. In 1919, the government tried to establish a national module of weights and measures, and a standardization of industrial production based on the metric system. The Japanese Architectural Academy also tried to change the *Kyoma* to a 2-meter module. But, during World War II, the Wartime Architectural Standard based on the *shaku* module was revived to protect the cultural identity of Japanese architecture. After the war, the Kenchiku-Kijunho (Architectural Standard Law) and the Kyotu-Jutaku-Kensetu-Shiyosho (Common Specifications for Housing Construction) were established by the government to legitimate the meter module in 1950. Thus, influenced by social and political conditions, the traditional *shaku* and the meter modules were each approved in turn.

Inspired by the translation into Japanese of Le Corbusier's *Le Modulor* in 1953 [Yoshizaka 1976], another academic committee for the creation of a national module was started in 1954. Their discussions focused on the following three points; whether to select the *shaku* module or the metric module; methods for industrialization and standardization of architecture; and the selection of the modular coordination. However, opinions about whether to select the *shaku* or the metric modules were highly diversified according to the members' occupations and interests. In the summary of these discussions, they agreed on a definition of the idea of a module,[5] but could not reach any recommendation for unifying the two modular systems. In 1955, the Jutaku Kodan (Institution of Public Housing) was established to supply public housing for the suddenly increased demand for housing all over Japan. Despite such conclusions by the committee, the Japanese government legitimized the JIS meter module, and prohibited the usage of *shaku* module in 1958. Nevertheless, people continued to favor the Tatami mat houses, and carpenters continued using the *shaku* module. Ironically, the industrialization and the standardization of houses in Japan from the 1960s progressed concomitant with usage of the *shaku* and the metric modules according to each element, and the government abrogated the prohibition of the *shaku* module.

Development of the ideas of minimum houses

In spite of the confusion in the creation of a national module, the ideas for the minimum house developed consistently in Japan after the Kanto earthquake in 1923. The members of a public institution Dojunkai, the Ministry of Interior, architects and scholars researched people's living and working conditions, and based on this experience and on the information from European housing policies and examples, they built public housing in concrete, and detached houses with traditional timber structures. In particular, information about the idea of standardized prefabricated collective housing in Germany, such as those of Walter Gropius in 1910, was introduced into Japan at that period.

During World War II, a number of prefabricated houses were designed. A Japanese architect named Ichiura Ken introduced the designs by Albert F. Bemis, and built 200 wooden prefabricated houses based on the Imperial module similar to those of W. Gropius's Trocken Montage Bau. In 1946, the Kogyoka-Jutaku-Seisan-Kyokai (Institution of Industrialized Housing Productions) was established to apply military technology to the housing industry, and a Japanese architect named Maekawa Kunio, a Japanese pupil of Le Corbusier, designed the first factory-made house "Premosu 7" using 1-meter wooden panels. As a result, by accepting the differences between the Imperial, metric and *shaku* modules mainly in their numerical senses, and without resolving their cultural contradictions, Japanese architects and contractors kept constructing prefabricated mass housing.

Jean Prouve's prefabricated minimum house

A Japanese architect named Ikebe Kiyoshi was one of the main members of the academy's national module committee, and continued the study of the module throughout his life. One of the reasons for his strong interest in the module was his involvement in the wartime project of prefabricated package housing from 1942 to 1945 at the Kokudo Kensetu Senso Kumitate Kennchiku Kenkyujo (Institution for the Construction of the National Wartime Prefabricated Architecture) by Sakakura Junzo. Sakakura had worked in Le Corbusier's atelier, and tried to translate French engineer Jean Prouvé's SCAL (Societe Centrale des Alliages Legers at Issoire) (fig. 1), made with lightweight steel frames and wooden panels, into Japanese traditional wooden structures.[6] In 1938 Prouve started to design prefabricated house for military barracks based on the portal frame system. In 1939 he collaborated with Charlotte Perriand and Pierre Jeanneret to design the first building with central metal portal frames and wooden panels, which was designed to be transported by truck and to be assembled with only two workers. The wooden panel was based on 3.3 ft. modules and interchangeable elements, and could be produced in different variations and sizes according to the designated function [Peters 2006: 36].

According to Nils Peters, even though his constructions are far advanced technologically for his period, his design procedure was a "tactile working method" [2006: 11] He was not interested merely in the technological aspects of the prefabrication, but more in exploring the expressive and economic possibilities and social significance of materials, which was only attained from his direct experience as a craftsman, and his dream for industrialization [Peters 2006: 17; Sulzer 1999: 11]. In contrast to Le Corbusier, Prouvé, as a furniture designer, was not explicitly interested in the ideology of human scale, and recognized that the beauty of the form derives from the expression of its process of construction. In his designs, the functions of architectural space and forms were defined by reflective communications between architects, builders and users.

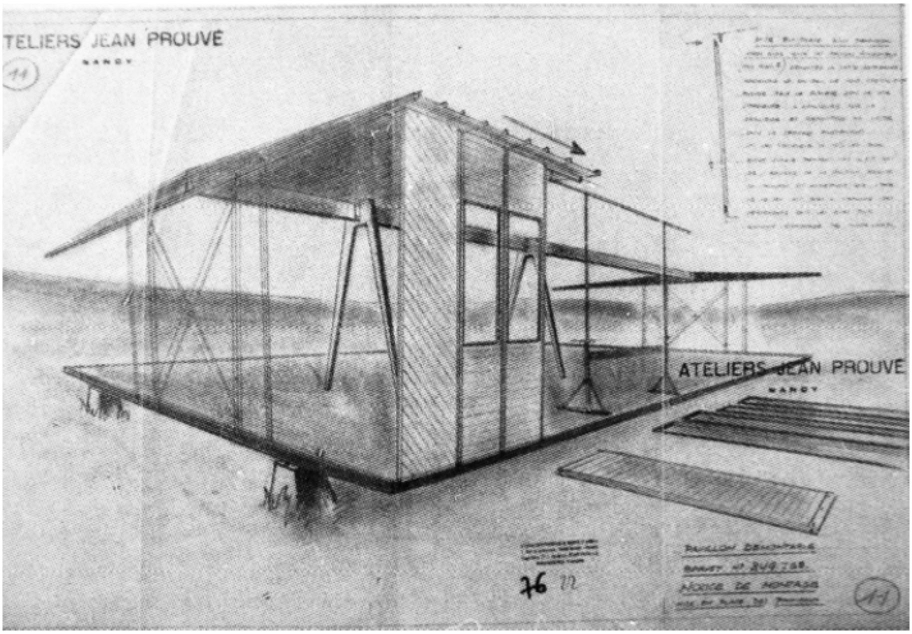

Fig. 1. Jean Prouvé's SCAL project [Sulzer 1999: 265].
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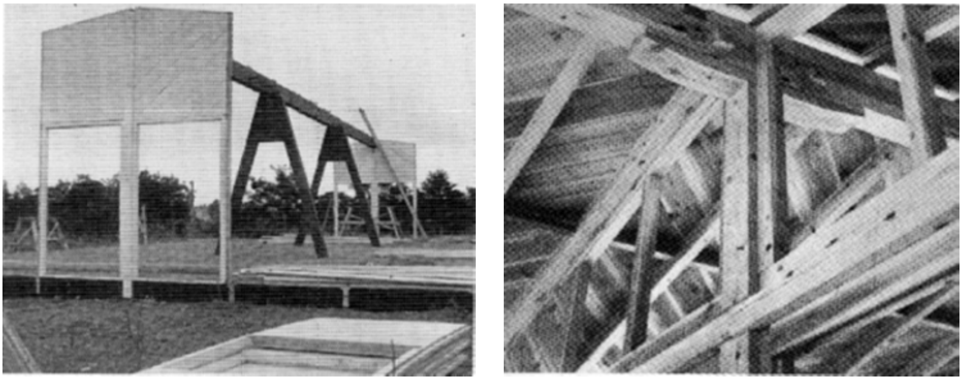

Fig. 2. Sakakura Junzo's applications of Prouvé's idea to a Japanese house, 1941
[Matsumura 1999]

Ikebe studied Prouve's model in detail, struggled with problems of its translation into a Japanese timber structure, and was apparently deeply influenced by Prouve's ideas and methods (fig. 2).

Ikebe's "Beauty is Mathematics"

In *Dezain no kagi* (Keys for design), Ikebe explained his initial inquiry about the ideas of module and prefabrication, and his approach to these themes:

> While I worked on the prefabrication of houses during the war, I encountered the most serious problem in the difficulty of understanding the inevitable human quality of space. Thus, after the war I decided to deny the hasty approach for industrialization of prefabricated buildings, and to work on how big a space a person actually needs in his life. ... This is a historical and universal theme, such as examined by a twelfth-century Japanese poet and by Le Corbusier. ... However, my approach was different from Le Corbusier. After pursuing the designs for these themes, I tried to return to the human standpoint to conceive these problems. In other words, I tried to examine how people can find the quality of space with architecture. Thus, I started to work on the idea of minimum volume with three dimensions instead of two, which the European modernist used [Ikebe 1979: 18].

He stated that architectural design has to begin by examining how small we can make the unit space for people, and is not about the appearance of form but about the realization of the desirable relationship between people and objects [Ikebe 1979: 22, 122]. Ikebe was opposed to his fellow Japanese architects' enthusiasm in following the ideas of Le Corbusier and other modernist architects, and instead, wanted to return to the real and qualitative nature of the relationship between man and his space.

He argued that "Beauty is Mathematics," because both exist with actual purposes and in people's cognitive and productive processes [Ikebe 1979: 94]. The beauty exists in the process of creating things, and mathematics is always implied in our act of finding solutions for problems. In his conceptualization of the module, he continuously explained that it is not merely the measurement but that it is derived from the proportion of the living space. He assimilated the unit of the module with the number 1 in mathematics as the primal element, and argued that both should be different in each culture. In his opinions for the ideas of module and designs of prefabricated housings, it is clear that Ikebe shared Jean Prouvé's substantial, humanistic, and process-orientated approaches.

Ikebe's critical standpoint was inevitably affected by his social situation in Japanese modernization after the war. In his article "Kenchiku no kindaishugiteki keiko" (Modernist characteristics in recent Japanese architecture) [1958] Japanese historian Inagaki Eizo wrote that Japanese architects had not critically examined their works during World War II, and retained their abstract idealism without addressing the reality of people's daily life. Ikebe continued to re-examine the problems in his wartime work and presented his study of the module as the new direction of architecture for post-war Japan. He also criticized Japanese architects' superficial preference for the aesthetics of traditional architecture. In the committee, he kept insisting on returning to the origin of the idea of module in rejection of the convenient compromise of the metric module, and on objectifying the traditional *shaku* module as one of the advanced aspects of Japanese architectural culture based on the material, form, technologies and people's lifestyle in the past.[7]

Ikebe believed that architectural standardization should not be aimed at merely for the sake of its industrialization, but should be accomplished by placing an emphasis on the complexity of human nature. In 1955 Ikebe described his intention in the study of the module as the foundation of freedom in designing, and as the key to reframing the idea of architectural design and production.

Numbered houses

Through approximately 100 house designs, Ikebe continually redefined his module systems. In the design of house No. 1 (fig. 3), he used a 3-m. module derived from J. Prouvé's work, and he tried to liberalize the design of the minimal house by carefully reevaluating people's activities in each space. In the design of house No. 3 (fig. 4), he used a 75-cm. module, which is a quarter of 3 m, and tried to apply this module in three-dimensional space by using it in the designs of desks, windows, stairs, passages and the restroom.

At this stage he compared his 56.25-cm. module, which is one element of a 75-cm. module (75 x 0.5 x 1.5), with the German Octameter module, and the *Kujira* (Whale) module for sewing Kimonos in Japan. He stated that they are all very close to each other in both the number itself and the coordination systems, which proves the universal validity of this module.

In the design of house No. 38 (fig. 5), which was commissioned by a Japanese publisher to introduce the ideas of the Case Study House in the American periodical *Art and Architecture*, he used an 8-cm. module based on his idea of the GM module (General Module: $X=2^n+2^{n-1}\ p_1+2^{n-2}\ p_2+2^{n-3}\ p_3$, ($p_1$, p_2, p_3=0 or 1)), which he derived from the measurement of peoples' living space. In this project Ikebe tried to prove that this module could be widely applied to the measurements of human movement, furniture, all parts and form of the house, and to its site plan on an urban scale. He explained that the new module would be effective in five ways: the joining of space, the change of spatial quality, the development of organic space, the separation and contrast of spatial structure, and the change of the constructional system of space [Ikebe 1995].

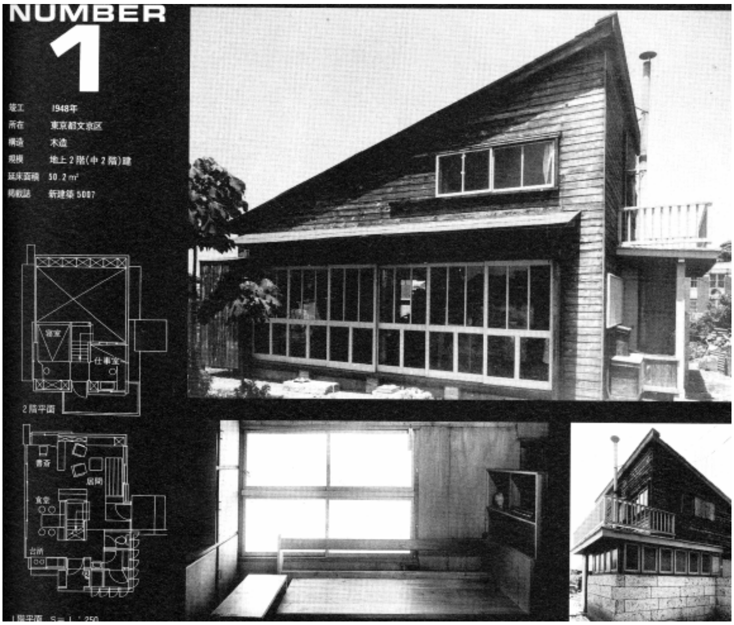

Fig. 3. Ikebe Kiyoshi's house No. 1 [Ikebe 1995]

Fig. 4. Ikebe Kiyoshi's house No. 3 [Ikebe 1995]

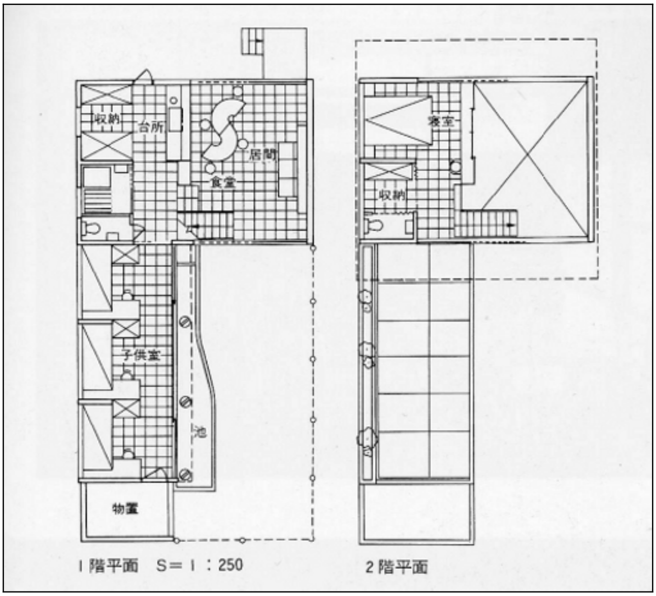

Fig. 5. Ikebe Kiyoshi's house No. 38 [Ikebe 1995]

This idea of the adaptability of the module system to the change of architecture in time shows his characteristic ideas of architectural function. He suggested that in modernity, according to the acceleration of the organization of human life, the idea of the function of space became too static. In other words, he conceived of architectural function as being formulated in response to the changes in people's lives.

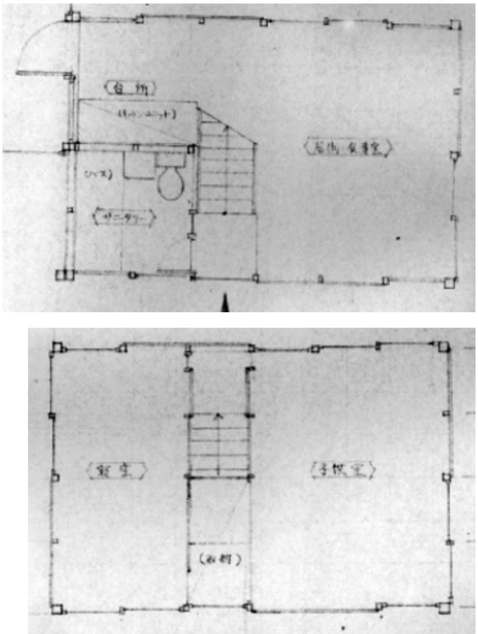

Fig. 6. House No. 71. First stage first- and second-floor plans (Owner's private collection)

House No. 71 (1962-1975) is one of the examples of his flexible house designs, which developed according to the change in people's lifestyle.[8] He designed this minimal house for a young family with two children. The unit module is 96 cm. and the original plan is a 6 x 4 rectangle in two stories (fig. 6). It is constructed with timber beam and frame structure with wooden panels. Even though it was a regular timber structure, the materials were all produced in factories, such as laminated wood beams and flexible boards, and the design was economically rational with bare structural beauty. The height of the interior space and all elements were designed with the module. On the other hand, he used highly industrialized kitchen facilities and bathroom equipment, which are disproportionately expensive compared to the other structural materials.

After an expansion of a back room in the second stage, and after the children grew up, in the third stage, Ikebe divided the second floor main room into two to provide individual rooms for the children with a top light, moved the stairs and kitchen to the back room, moved the bathroom to the second floor, and added the second floor on the back (figs. 7-9). The validity of the module was continually re-examined in these transformations, and he applied different design methods and elements to resolve the problems and to create new usage of the spaces in order to respond to the changes of the family's lifestyle. The husband and other family members kept struggling to modify and maintain the details and space in order to adjust the environment to their needs and feelings, collaborating with Ikebe to make it their own.

As he attached importance to the flexibility of life space and production systems, his designs of houses were based on simple structures and on the flexible arrangement of plans. He gave serious consideration to the engineering systems in the kitchens and bathrooms, and tried to rationalize the housewives' working place as much as possible. As Nanba Kazuhiko, one of his pupils, stated in his book on Ikebe, Ikebe tried to force clients to inquire about the appropriateness of their habitual lifestyles in that logically designed space, and let them work on their houses by themselves [Nanba 1999]. Thus, in such dialectical communication with clients in the actual examination of the scale and engineering of the house, Ikebe examined the communication methods in the design process.

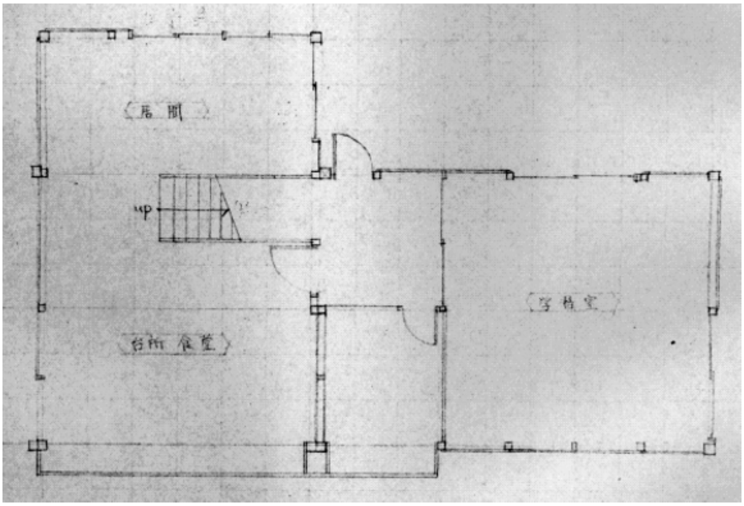

Fig. 7. House No. 71. Third stage, second-floor plan (Owner's private collection)

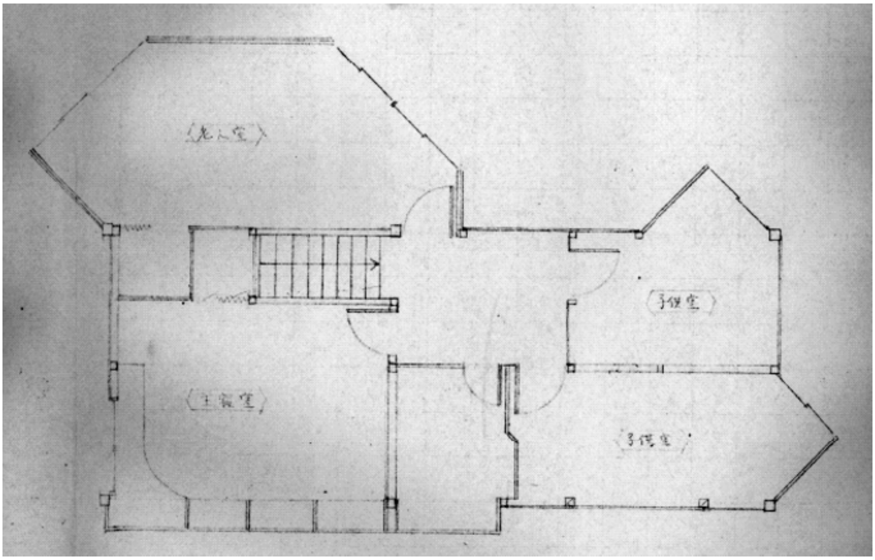

Fig. 8. House No. 71. Third stage, first-floor plans (Owner's private collection)

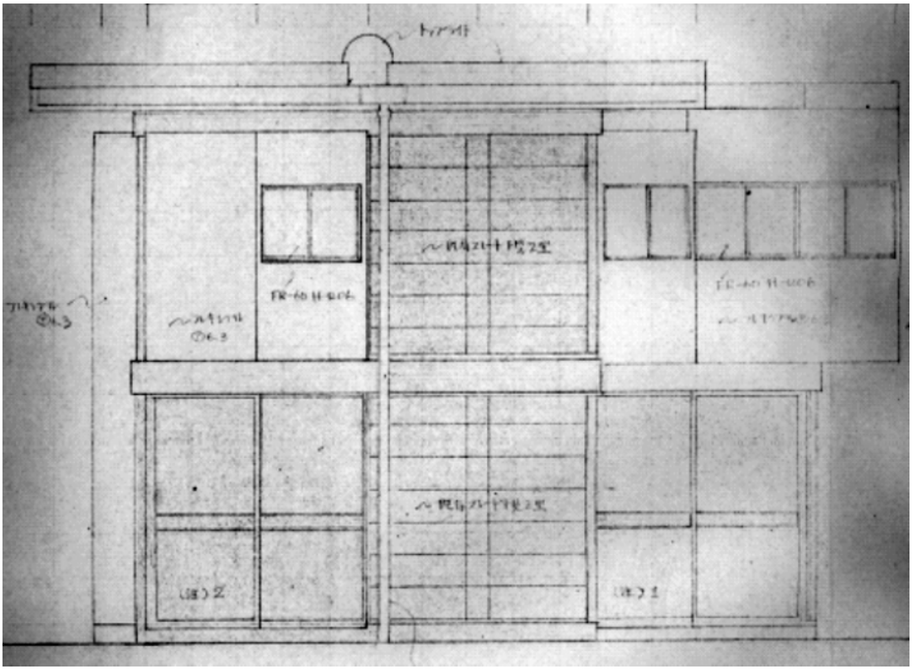

Fig. 9. House No. 71. Third stage Elevation (Owner's private collection)

System theories and the idea of module as a flexible mathematics for creation

In the last stage of his career, Ikebe presented various ideas of design system theory, which were later developed in the ideas of space syntax by his pupils. Two of them, the modular coordination on an urban scale, and Design Sugoroku (Parcheesi), are essential for understanding their relationship with his idea of module:

- The modular coordination on an urban scale: In his article "Ningen-shugo-wo-kisoni"(Considering architecture based on the nature of human gathering) [Ikebe 1979], Ikebe stated that we have to conceive housing space as interrelated to its outside space, and created a diagram showing an order in the scale transitions from individual space to the whole environment. He argued that by conceiving the idea of dwelling from the nature of human gathering, we would be able to design architecture from a wider social viewpoint (fig.10).
- Design Sugoroku (Parcheesi): Ikebe conceived this diagram as the correlative network of the factors in design. He specified ten elements; Work, Cost, Tradition, Purpose, Appearance, Material, Distribution, Environment, Function, and Standard, and grouped them into three categories: those which relate to the users, producers, and social conditions. Through examining any one of these elements, the idea would develop into a substantial design. By passing through the centered element of Standard, the development of ideas and the collection of information will be systematically organized to facilitate teamwork in architectural design (fig.11).

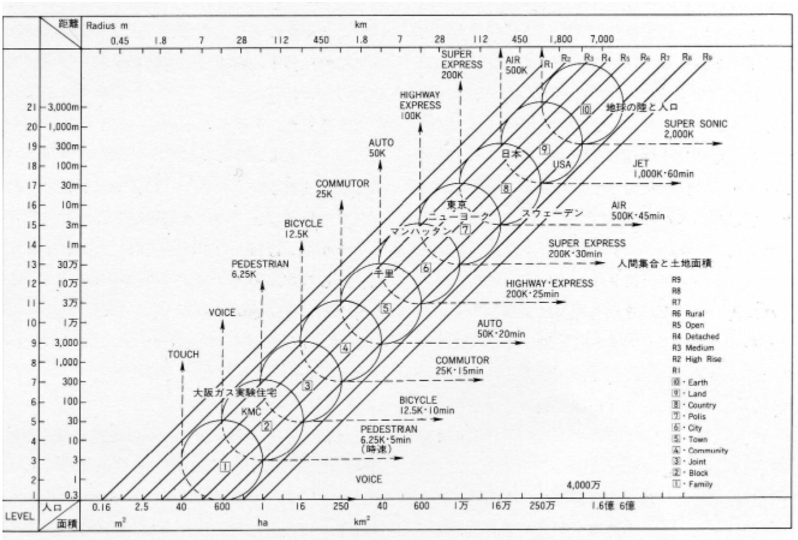

Fig. 10. Ikebe Kiyoshi's conceptual diagram: The modular coordination in urban scale [Ikebe 1979]

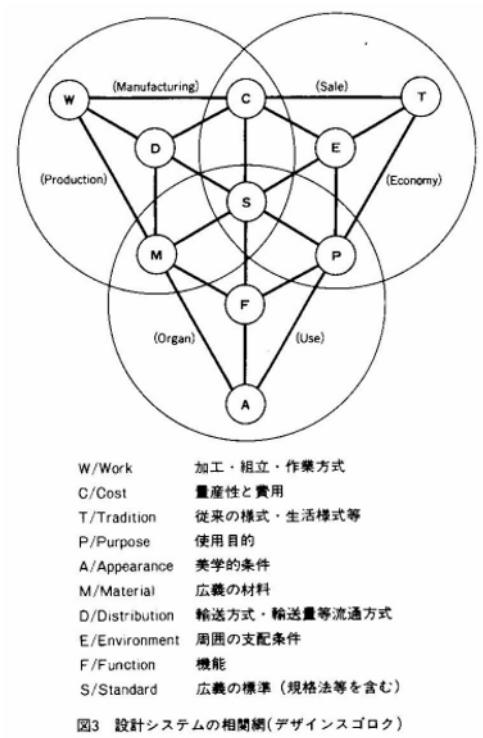

Fig. 11. Ikebe Kiyoshi's Design Sugoroku [Ikebe 1979]

These diagrams represent Ikebe's intentions to extend the ideas of the module and standard into a general system to make all information accessible to all participants. He thought that the psychological and social factors of architecture could be studied more clearly and qualitatively by organizing the relationships between human and environmental factors in geometric diagrams. In other words, these design system theories are synthetic systems of communications based on geometrical orders. In these theories, Ikebe cultivated four approaches:

1. putting a lot of emphasis on feedback from practice into theoretical studies;
2. creating new flexible logical structures for the design process;
3. trying to develop a method to incorporate the ambiguous factors of human life; sensual and habitual, into the idea of architectural performance;
4. reinterpreting the idea of module and standard as the communal base for collaborators to answer the economical, technological and social demands.

Conclusions

As explained above, Ikebe Kiyoshi was consciously opposed to Le Corbusier, and presented his idea of module from his different viewpoint and interpretation of mathematics and architecture. He focused on its social, cultural and physiological meanings, attempting to return to the primal inquiry about man's relationship with his environment. For him, mathematics is not an abstract form or number but something both substantial and metaphorical, capable of organizing human thinking and creation, and which should clarify the cultural significance of Japanese people's living space beyond the ideology of Westernization. Ikebe's attempts to reinvent an idea of the module was also the rediscovery of the role of the architect in taking and inventing a balance between multiple and flexible human and social factors in architectural production.[9]

In 1958 renowned Japanese architect Isozaki Arata criticized Ikebe's ideas, saying that the conflict between architects and clients cannot produce a standardized solution for housing, and the mainstream of housing production would be dominated by identical housing determined by the bureaucracy and with readymade houses by the housing industry. Isozaki also envisioned that outside of these domains architects would only design irrational houses.[10] Actually, in the history of Japanese modern architecture, Ikebe's designs have been mostly appreciated for their rational and logical approaches without considering his critical consciousness of modernizing society and his presentations of alternative ideas for the relationship between mathematics, beauty, technology and architecture.

Since Ikebe's death in 1979, the prefabricated housing market has expanded dramatically to answer the need all over Japan for high technological performance, and comfortable living environment.[11] The developments of visual technologies are transforming the relationship between our perceptions, forms, architectural space and production [Nanba 2007]. Such phenomena raise serious questions about the humanistic foundation of architectural creation, and the social and cultural meaning of our living environment, and the necessity to reexamine the meaning of the unification of human scale, forms and productions of architecture. Through examining the process used by Ikebe to create the module, and the prefabricated and standardized minimum house, we can see that Ikebe's ideas were not only embedded in his social and cultural consciousness against the modernization of Japanese architecture, but also presented alternative tactile, substantial and flexible approaches for its realization. This analysis of Ikebe's ideas and works argues that, even though Ikebe shared the modernist ideology and dreams in mathematical rationality and industrial technologies with Le Corbusier, he conceived different frameworks of their relationship with people, and, beyond Isozaki's cynicism, explored further possibilities to respond to the problems of architectural production of our time.

Acknowledgment

This paper is based on the author's oral presentation at the annual conference of the Society of Architectural Historians in April 2007.

Notes

1. For example, among the developing countries in Asia, the deliberation of public housing for economically underprivileged people is a most urgent social issue. The greatest difficulties related to the discussions of housing concern their close relationship with the social, economic, cultural and technological problems, and with these multidisciplinary problems, the ideas of standardization and module for architectural production have been discussed as one of the key issues.
2. There are two other scales, Sun and Ken for this module.
3. Kiwari deals with the basic arrangement of columns and beams according both to the functional needs of buildings and to the structural balance and strength of members.
4. Hashirawari determines the styles of buildings, their interior arrangements, ornament and other architectural fixtures.
5. The committee decided on this definition of module, "Module is the basic element of architectural measurement, which gives a certain order in the usage of the measurement in the entire area of architectural production; the production of architectural materials and parts, as well as the whole aspect of design and construction, and it enables the mass production of architecture."
6. Jean Prouvé started his career as a blacksmith and craftsman, but extended his creative involvement to the design of prefabricated buildings. In 1930 he was involved in the group UAM, which was dedicated to establishing a connection between art and industrial production, with Le Corbusier, Charlotte Perriand, Pierre Jeanneret and others.
7. In his doctorial thesis on the idea of module [1961], Ikebe explained that European modernist architects conceived their ideas of module inspired by Japan's traditional *shaku* systems, but that Japanese people had been so used to its advantages that they are now learning from those European ideas and having difficulties in creating a new module. See [Nanba 2007].
8. This house is author's parents' house, and this section is based on my actual experience in eighteen years as a resident.
9. Ikebe briefly explained how he was challenged to resolve contradictory problems in defining a module system as a condition for flexible designing. See [Ikebe 1955].
10. Furthermore, one of the limitations of Ikebe's houses was that they often cost more than other public housing and readymade houses.
11. Prefabricated houses now dominate Japanese urban and rural landscapes, ignoring their local cultural identity. They have also changed from an inexpensive to an expensive, tailor-made commodity.

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Research

Can Chaos Theory Explain Complexity In Urban Fabric? Applications in Traditional Muslim Settlements

Abstract. The present work is a limited analysis of the traditional urban fabric of Muslim cities in the light of this theory. Chaos theory provides better instruments for the analysis and understanding of the traditional urban fabric in old Muslim cities that have, paradoxically, long been considered as lacking order and thus "chaotic" in the pejorative sense! Beyond the analysis, our study also aims at providing a fresh approach to the built environment that shifts the architectural and planning professions from the traditional design and planning approach to a self-generating process that, once set up, would function and develop without need for intervention. However, the present study will be limited to the application of such concepts at two levels. At the urban level, it will provide evidence of the existence of chaos and fractals at the scale of the city. At the domestic level, concepts are applied to housing evolution and community spaces.

Introduction

Despite the increasing interest of scientists from various fields of physical, natural and human sciences in chaos theory, its application in urbanism and in architecture seems to be still in an early stage. Among its earliest applications in architecture and urbanism are those of Batty [1990, 1994], Bechhoefer and Bovill [1994, 1996], Bechhoefer and Appleby [1997] and Salingaros [2003]. In a further step, Crompton [2001] attempted to use the fractal dimension in measuring the complexity of the dwelling space through the counting of the number of places it offers to users in an indoor space. However, in most of these studies the mathematical instruments seem to overwhelm the new approach and thus limit the scope of architects and planners to explore it in both analyzing the built environment complexity and in adopting it in their practice, a fact that was also noticed by Ostwald [2001: 79] and Mulligan [1997]. In contrast to the pessimistic standpoint of Ostwald [2001] announcing the death of fractal architecture in the late twentieth century, this paper presents the thesis that "chaos and fractals" is a dormant theory and has hidden potential that is still valid for exploration. Since its beginnings, it has been abused by architects, initially to justify their bizarre forms, then to legitimize their irrational actions [Salingaros 2006; Ostwald 2001]. To date it is still misused by considering it as a design fashion. The latter approach, mostly based on aesthetic considerations, results in rigid forms and elevations that show some characteristics of fractals and complexity but which nevertheless clash with the essence of the theory, which aims at reinstating the natural dynamics of reality in architecture and urbanism.

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Application to traditional Muslim cities

The present work is a limited analysis of the traditional urban fabric of Muslim cities in the light of this theory. Chaos theory provides better instruments for the analysis and understanding of the traditional urban fabric in old Muslim cities that have, paradoxically, long been considered as lacking order and thus "chaotic" in the pejorative sense!

Beyond the analysis, our study also aims at providing a fresh approach to the built environment that shifts the architectural and planning professions from the traditional design and planning approach to a self-generating process that, once set up, would function and develop without need for intervention.

Chaos and fractal theory relies on a series of concepts that establish its structure and define its methodology and techniques. Among the concepts that are directly linked to the physical environment are growth and self-similarity, randomness and unpredictability, dynamism and non-linearity, order and disorder.

When considered in light of this theory, the city has many scales, from the largest regional scale down to the scale of the geometric pattern of ornaments [Batty 1994; Salingaros 2006, 2007]. However, the present study will be limited to the application of such concepts at two levels. At the urban level, it will provide evidence of the existence of chaos and fractals at the scale of the city. At the domestic level, concepts are applied to housing evolution and community spaces.

Chaos: from disorder to a highly hidden order

Most of the studies on Muslim cities were initiated by Western scholars during the nineteenth century as a result of Western colonial expansion to overseas and the spirit of exploration that characterised European intellectual life. The extensive field and theoretical research undertaken, although mostly objective scientifically, was paradoxically motivated by the military and political agendas of the colonial powers. Thus many value judgments spoiled the results of the research and generated skepticism in later generation of non-Western scholars [Said 1979]. Two of the sources of these misjudgments were the glorification of early culture and reference to the Greco-Roman model of cities. The Western scholars' approach, already distorted by the effect of superiority and the reference to Greeks and Roman legacy of planned towns, projected the Muslim cities on a background that was significantly shaped by Cartesian geometry. A pattern of streets following straight lines and right angles, reflecting the principle of orthogonal grid organised around the Cardo-Decumanus axes, was thus considered as the golden rule for measuring the degree of clarity and order in other non-Western cities [Morris 1994: 55-59; Sauvaget 1949; Eglash 1999: 3]. The complex urban fabrics of Muslim cities was inevitably considered to be a reflection of disorder. This disorder was then explained at the cultural, social and political levels as a sign of absence of citizenship in the Muslim societies, and lack of administrative and political institutions [Sauvaget 1949; Marcais 1957; Letourneau 1985].

According to Batty [1994: 12], this view was not limited to a few non-Western scholars, but generally characterised the philosophy of the nineteenth century, based on the mechanistic knowledge, Euclidean geometry and reductionism which together formed the cornerstone of the new sciences. Often the norm was to impose the geometry of the straight line on forms that clearly did not meet such geometrical purity in reality [Batty 1994: 19].

This approach continued to shape our present cities through the domination of perfect geometry on urban planning and architecture. Irregularity of form conflicts with architects' and planners' intuition and predisposition to think in terms of the simplistic geometrical order based on the geometry of Euclid and the Greeks [Batty 1994: 8].

It is only recently that irregularity in geometry is considered as a sign of complexity and richness rather than as simply a lack of order. Organic cities, mainly characterized by their irregular geometry, are cities whose morphology is governed by a deep underlying structure.

Chaos theory, also known as the science of complexity, provides efficient instruments for uncovering this underlying structure. Irregularity should therefore be analysed as the result of a set of rules that governed the successive physical events that gave shape to the city, which accrued in layers over time. The factor of time, that is the age of the city, is thus present in its morphology through the intensity of events and defines the degree of urban complexity.

Chaos applied to Muslim cities

Initially, Muslim cities could be classified into four main categories. The first category is religious (Makka, Medina, Jerusalem). The second is royal (the Round City of Baghdad in Iraq, Al-Zahra in Muslim Spain). The third is inherited (Damascus, Aleppo, Cherchell in Algeria). The fourth is spontaneous (the case of most other cities). The religious and the spontaneous ones generally develop as a natural consequence of the physical conditions of site and the small actions of settlers in the absence of any top-down planning, while the second ones are generally shaped as a consequence of a political motivation for their foundation. Most often the structure of the latter is highly defined by a simple and clear geometry. The Round City of Baghdad, founded by the Abbasid caliph A. J. Al-Mansoor in 764, is a case in point [Ben-Hamouche 2004] (fig. 1).

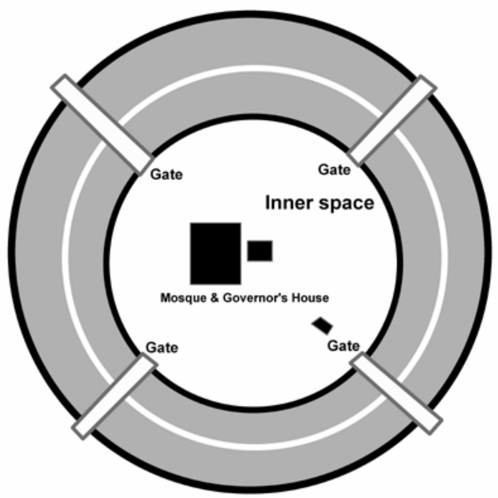


Fig. 1. The Round City of Baghdad, showing the strong geometric pattern. Drawing by the author

Regarding the third category, inherited cities, due to historical and geographic circumstances, many Roman cities were inherited by Muslim conquerors and thus were subjected to many transformations. Their initial grid layer gradually disappeared under the irregular geometry of an incremental development process.

Regardless of their origin, in the long run urban fabric acquires a high level of complexity in which the initial urban form is turned into a pale background for later development. By stripping back successive layers, we can infer that during their foundation and early stage of development, they were relatively simple and had a certain geometric clarity. Over centuries, they ended up with a complex urban fabric characterised by a high degree of interlocking of buildings, a variety of architectural details, and an irregularity of street patterns. The Euclidean geometry that might have explained their initial forms can no longer provide instruments to deconstruct their present complexity. The theory of chaos and fractals, that is, the mathematical and geometrical study of irregularity, provides an alternative theoretical framework for the analysis of this complexity [Bovill 1996b: 3].

Examples of Muslim cities

Damascus and Aleppo provide good examples for the distinction between the two types of geometry and the evolution from a previous to the later one. The grid pattern of road networks that were established during the Hellenistic era dominated the initial urban tissue. After the Islamic conquest, the city structure was gradually transformed into narrow winding streets, dead-end alleys, and interlocking buildings. In reconstructing the old urban fabric, Sauvaget [1949], who might have been highly influenced by the French mathematicians' rigour, explained the radical transformations as a consequence of change in the institutional system, and lack of control. In other words, it was interpreted as being due to an absence of order.

In the light of chaos and fractal theory, the initial Euclidean geometry, imposed by the strong politico-military power of Romans, was substituted by the myriad of individual decisions at a much smaller scale that generated a fractal geometry and reflected the reality of daily life [Batty 1994: 8].

The strong grid that characterised the initial layout of Damascus and Aleppo did not prevent these cities, after centuries of development, from hosting complexity and thus becoming similar to the other cities such as Baghdad, Cairo or Algiers.

However, time is a major factor in defining the "maturity" of a city and determining its degree of complexity. An older city is always a recipient of many physical events and accretions that have piled up over each other. Its "fractal dimension", even without mathematical calculations, could be measured by its age, which goes back to its date of foundation. For instance, Manama and Muharraq both emerged in 1780s, and are very young if compared with Cairo (950 A.D.), Baghdad (764 A.D.) and Tunis (7th century A.D.). Inversely, we would expect to find that Medina (622 A.D.) and Mecca (2000 B.C.) are more complex than any other cities (fig. 2).

Fig. 2. A portion of Manama city, Bahrain, in 1950s showing the low intensity of urban space usage compared with older Muslim cities.
Source: Directorate of Survey, Bahrain

The evolution of the city of Kufa, founded in 637 AD, is another example of the divergence between the two types of geometry. Despite the absence of initial plans, there is an account that gives us the details of its foundation. Describing the process of its layout, historians state that a centre was first established as a square that was defined according to the shot of an arrow to the four horizons, that is, approximately equal to 480x480m [Al-Tabari 1963; Aal-Mawardi 1960; Jait 1986]. In a second stage, roads were laid out according to a set of norms that were ordered by the supreme ruler, the Caliph [Al-Tabari: II, 17]. Accordingly, main roads (*manahij*) were to be forty cubits; those following the main roads, thirty cubits; those in between, twenty; finally, lanes (*aziqqah*), seven. Planning units called *khitat* (plural of *khitta*), to be developed as residential quarters, were also defined within this road network. In describing the development process, historians did not mention any intervention of the ruler in organising these units. As part of the collective property of tribes, it seems that these plots were subdivided among members of tribes and developed solely by individuals. Thus construction would have been carried out in an incremental process. It can therefore be deduced that the city layout initially had a Euclidean geometry based on the road network and the central square. However, this geometry would have been modified gradually through the incremental process comprising the private and collective actions into a fractal geometry.

Two aspects of fractal geometry could be made evident in this process. The first concerns the road layout. A tree-like pattern of roads in a hierarchic graduation of four levels from the main arteries to the narrow streets has been established. Two other levels that consist of semi-private and private paths often occur over time. Blocks that are developed out of the urban *khitat* units often comprise winding streets and dead-end alleys. In some cases, dead-ends emerge in response to the need to provide access to deeper shares within the residential block after subdivision into smaller shares. In other cases, they are formed from the blocking of one end of a semi-private street and the addition of an arch or a gate, an action that is often undertaken by neighbours for reasons of security and privacy. This might explain the high percentage of dead-end streets that characterises old Muslim cities, ranging from 25% to 45% of the total network [Raymond 1985].

In fractal jargon, this process could be simulated by the continuous iterative process of generating lines. The main roads initially laid could be considered as the "active lines" [Eglash 1999: 16]. Soon they become "passive lines" through the steps of replacement. Theoretically, this process can continue endlessly. However, in the case of the urban space in old Muslim cities, it might end after the sixth or seventh steps of the down-scaling process, which correspond to thresholds of houses and rooms. This process is explained in fig. 3.

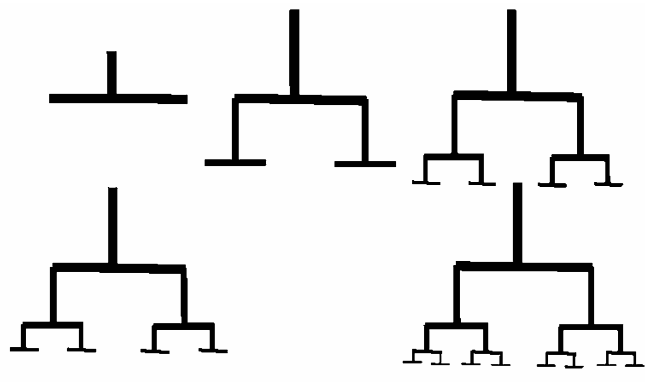


Fig. 3. The generation of road pattern adapted to Cantor's model of fractals

The geometry of the main roads was also subjected to a continuous process of reshaping though the encroachment of private houses on public space. Despite the *Hisba*, which permanently controlled all public spaces and buildings for the purpose of removing objects that obstructed the flow of traffic, high overhead projections and extra steps in front of doors were often permitted or passed unnoticed, and thus increased the fractal shape of the roadsides. The osmosis-like relationship between the *Muhtassib* and the proprietors of houses along main roads thus dictated the irregular landscape of the roads and generated their urban architecture (fig. 4).

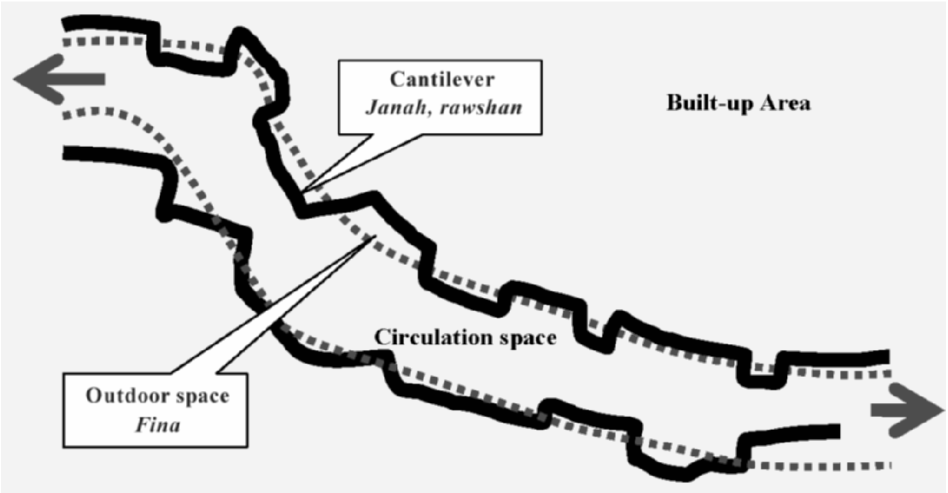


Fig. 4. Street form shaped by the osmosis-like relationships between the flow of traffic and individuals' encroachments

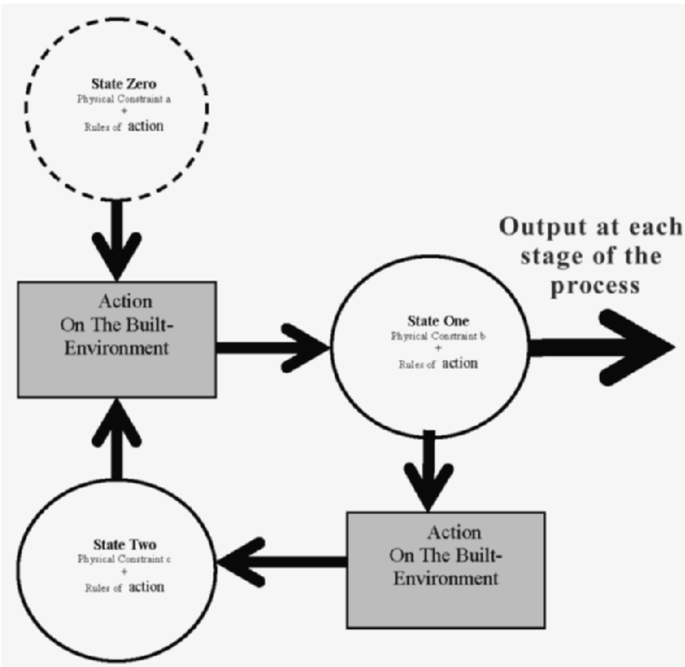


Fig. 5. Application of the Cantor principle to action in the built environment

The sections that follow will describe the sources of complexity in traditional Muslim cities, based on the Islamic principles that govern the actions of the individuals and the communities.

Sources of complexity in traditional Muslim cities: norms and behavioural standards

Complexity in Muslim cities results from the incremental process of development and the slow urban growth that develops out of small actions. Two interacting factors constitute this process: the time factor – i.e., development occurring in successive stages – ; and the intensity of individual actions in the space. This seems to be the origin of any other so-called spontaneous, natural and unplanned developments that generate urban fabric in the absence of official interventions, voluntary design, or the guidance of professionals [Batty 1994: 8]. However, in Islamic cities the process was ruled by a set of endogenous norms and behavioural standards that stemmed from the symbiosis between Islamic jurisprudence and the specific conditions of each location. Unwritten codes that structured this generative process were thus established in each region and city [Hakim 1994, 2007a, 2007b]. Fig. 5 presents the model for fractal generation adapted to the individual actions on the built environment, a process that will be examined further at the end of this study.

We will now elucidate some of these codes that generated complexity and thus help decompose the fractal geometry underlying the traditional urban fabric in old Muslim cities. Since housing forms the largest part of the city space, the analysis will be limited to the domestic scale ranging between the neighbourhood and the housing units.

Fig. 6. Portions of urban fabric. a, above) in Muharraq, Bahrain showing the mass-void ratio and the different plots forms; b, right) in Tunis, showing the variation of houses at a more detailed level. In spite of the difference of locations, they show a striking resemblance due the similarity of "the rules of the game"

Private legal power

Private legal power is called *Wilaya*, an Arabic legal term that connotes the freedom to act on one's property without the consent of any outside party. In Islamic jurisprudence, a person in full possession of his mental and physical capacities has the right to act on his own property without need for permission from the ruler. For instance, a person has a right to open a door, seal a window, add a partition wall, relocate an entrance door as long as no damage is caused to neighbours or to the public. This is the code behind the large freedom of action that people in traditional Muslim cities enjoyed and that was the source of the uninterrupted flow of small actions. Over time, these small actions generated a cascade of details and a fractal geometry that could be seen in the form of urban fabric (figs. 6a, b).

A principle of reclamation of un-owned or dead land constitutes a large margin of freedom to individuals. The majority of Muslim jurists agree that a person could reclaim this type of land through building, farming and any other action without referring to the authorities. This applies mostly to vacant land that is far from urban concentrations. However, unused spaces within the cities such as "overhead air" in the streets that is beyond the height of passers-by is also considered as dead space, a fact that gave rise to the projections and arches-likes rooms over streets.

Agreements and right of precedence

As an extension of private legal power, inhabitants and neighbours in traditional settlements had the right to establish different agreements among themselves regarding the mutual use of their properties and exchanges of rights. For example, a person could sell part of his wall, or rent a right of support or a passage to his neighbour for a defined period of time. Most of these agreements were verbal and were established in the presence of other neighbours as witnesses. They were also transmitted orally from fathers to sons over centuries.

The right of precedence meant that the rights of any pre-existing buildings on the site prevailed over those of buildings built later. In developing their plots or undertaking a new action, proprietors either had to consider the existing ones in terms of privacy, shade, wind, etc., or to establish agreements to overcome the constraints on his action. When positioning a window or an entrance in his new construction, a proprietor had to avoid placing them opposite the pre-existing ones of the neighbours facing him. In this sense, any new action becomes a problem-solving process involving a set of pre-existing constraints. Therefore, over time agreements and rights of precedence became accumulated conditions that generated specific constructive and architectural solutions and thus further generated fractals.

Inheritance law

In Islamic law, heirs are classified into categories according to their kinship and sex. Basic shares stated in the Qur`an are as follows: 1/2, 1/4, 1/8, 1/3, 2/3, and 1/6. For instance, the property of a dead man who left a wife, two girls and a boy should be subdivided so as to give one-eighth of the property to the wife, and the remaining seven-eighths to the children. Since a boy is entitled to twice the share of a girl, the remaining property will be subdivided into three parts, with the boy having one-half (that is, 7/16 of the whole property) while the girls will have one-fourth each (that is, 7/32 of the whole). Following the death of the original heirs, the same process will apply on each of the shares depending on the number of descendants of each heir.

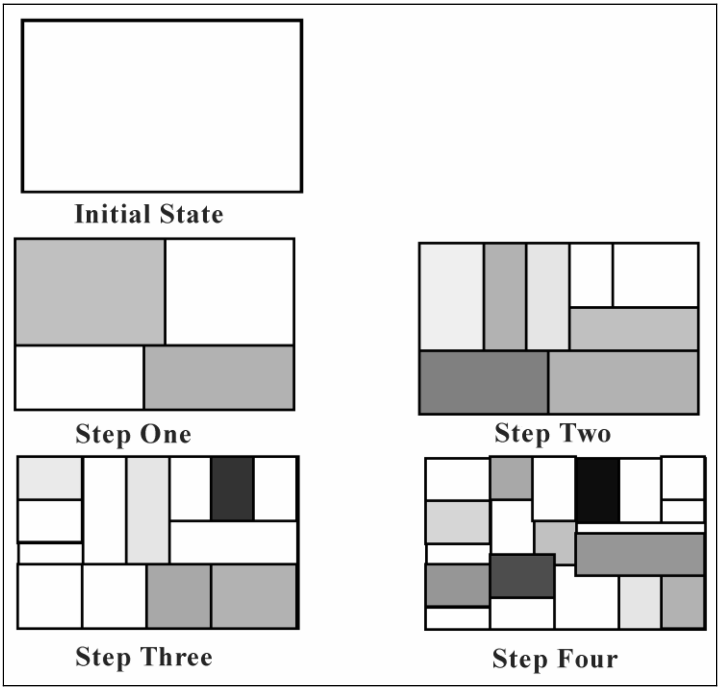


Fig. 7a. hypothetical process of land subdivision according to Islamic law

Fig. 7b. The impact of land subdivision according to Islamic law on the urban fabric of Algiers

Within a century, the same initial property might undergo up to three subdivisions, resulting in a cascade of small functional shares of urban spaces.

Describing the Islamic law of inheritance, Brugman [1979: 88] states that it "is characterised by an excessive fragmentation of the estate". However, jurists placed conditions on the subdivision process that preserved the usefulness of resulting shares in terms of conditions of use, accessibility and independence. Regarding the mechanisms and rules of subdivision, Akbar [1988: 135-136] thoroughly discussed the divisibility of elements and the principles of subdivision. For instance, some elements in the estate, such as wells, small courtyards, stairs, passageways and baths, could hardly be subdivided and were thus either kept intact and used in common, or were sold entirely if that was possible.

These mechanisms of subdivision generated continuous changes in the boundaries of properties and locations of partition walls, and were thus continuously incrementing the complexity of the urban fabric and increasing the "fractal dimension" of cities. Property lines and party walls in any given block increase in number at each stage of subdivision. The older the city is, the more it contains smaller shares, and shorter and irregular lines, and thus the more fractal it becomes (fig. 7).

Pre-emption

Pre-emption is the priority given to the co-owner in a shared property to substitute himself for an outsider purchaser, if the other co-owner decides to sell his or her share, in other words, the right of objection. According to this principle, shareholders in a collective property that could not be subdivided have the right to prevent each other from selling their shares to an outsider. This principle had the opposite effect of the subdivision that results from inheritance law, in that it permits the reunification of the smaller parts into a new, larger property. Further, if there is a large number of co-owners, the right of pre-emption is granted to all of them, even if their shares were different. In this case, the share that was to have been sold is to be further subdivided wisely and equally among partners who wish to avail themselves of their right of pre-emption, provided that this is feasible.

Scholars of different schools of Islamic law have diverging points of view regarding the extent of this principle. According to the *Hanafi* school of law, this principle could be applied even to neighbours. Based on the theory of prejudice, the right of pre-emption was granted gradually to the full partner, then to the partial associate(s) who shared some services such as an entrance, or a passage, and finally to the neighbor who has an adjacent property or lives in a same lane. If all those with the right of pre-emption resigned, the right could be granted to the back neighbour, who might have an entrance in another lane, and shared a back partition wall with the property to be sold. In the absence of the legal process, this right is often conceded to neighbours in order to maintain good relationships. In physical terms, the principle of pre-emption unified fragments of an earlier property in new plots and thus generated greater complexity in the geometry of the block. In case of two houses on opposite sides of a street, they can be joined by an overhead room or bridge-like passage that maintains fluidity of circulation yet allows connection between the two houses (fig. 8).

The principles discussed above do not form an exhaustive list, but are only examples. Many similar codes that have a direct impact on the generation of "fractality" could be established through an exhaustive study of Islamic law.

Complexity and "fractality" in old urban fabric were the result not only of these principles acting separately, but also simultaneously and in parallel. Consequently, transformations of the site took the form of a sedimentation of successive layers of actions and changes effected by inhabitants over time. Therefore, the older the urban fabric, the more complex its geometry.

Fig. 8. A street in Casbah, Old Algiers, showing an overhead projection linking two opposite houses

The language of chaos and fractals applied to Muslim cities

It is not possible to dissociate chaos and fractals. Due to their interaction, they are either stated together [Crilly et al. 1990; Strogatz 2001], or somewhat interchangeably [Gleick 1988] For instance, according to Kenkel [1996] the fractal dimension can be viewed as a relative measure of complexity, and thus of chaos, while fractals are presented as part of chaos theory [Gleick 1988].

Literally speaking, chaos is a broad term for indicating the science of complexity. It applies to natural sciences such as physics and biology, as well as to human sciences such as economy and sociology, whereas fractals are limited to the geometry that is concerned with the application of non-Euclidean analysis, measurements and calculations to complex objects. The two terms are thus used together to explain complexity in real life and describe the inherent irregularity of natural objects.

The "theory" of chaos and fractals has been described as a collection of examples, linked by a common point of view. Its properties are not formally defined and it is thus not an organized theory [Lorimer, Haight and Leary 1994]. Eglash [1999] defined some of the essential properties, which are recursion, scaling, self-similarity, infinity and fractal dimension.

Now we will select some of the key properties and verify their presence in the urban fabrics in traditional Muslim cities.

Growth and self-similarity

According to chaos theory, growth is not merely an increase in size, or a zooming out of a body to another scale. Rather, it is an exponential multiplication, or reduction, within the system and a successive passage from one scale of complexity to another. Zygotes are not merely scaled down human beings, and the process of ontogenetic development is far more interesting than mere enlargement [Gleick 1988: 115].

One of the first models for grasping growth was the experiment of measuring the exact length of Britain's coastline. Mandelbrot [1967] observed that as the scale of measurement becomes smaller, the measured length of the coastline increases infinitely. Although the total area is finite inside the curve, the polygon, the boundary line of the shape itself could have an infinite length. This was also explained by the Koch curve.

In traditional housing, the principle could be applied in observing the growth stages of the old cities. For instance, the city of Algiers had a fixed area of 45 hectares within the city wall during the three centuries of the Ottoman period due to site constraints. However, it witnessed a significant variation in its population size and thus, its density grew from 50,000 people to 100,000, and then reached 150,000. At the beginning of the nineteenth century, it shrunk to 30,000 due to epidemics, economic recession and wars. Such fluctuation in population size, despite the size of the site being fixed by topographic constraints, shows the "plastic" capacity of urban space to absorb endogenous growth.

In many cases the stages of urban growth are often reflected in the shift of the cities' external walls. Ghardaïa (south of Algiers), was not extended until all of the urban space inside the city wall was entirely consumed. Suburbs outside the city wall were seen as a negative form of urban development that was often avoided because it was hard to defend in case of attack.

Self-similarity at different scales of a system is another principle of chaos theory. A self-similar object is one whose component parts resemble the whole. In growing, chaotic objects tend to reproduce themselves at different scales while maintaining the same structure at all levels of growth. Self-similarity is therefore different from sameness. It is rather a resemblance in terms of structure, as well as in the process of ontogenetic development.

This principle can be seen in three forms of traditional housing that reflected the different scales of the physical environment. At the *city level*, housing was characterised by its high degree of compactness and the predominance of areas covered by buildings over open spaces [Ben-Hamouche 2008]. Ghardaïa had only a single public space in the city that is still used as a weekly market and a daily meeting place. At the *neighbourhood level* another unique area could be found as a community space in each compound. On describing the foundation of Kufa (637AD), al-Tabari (died 926 A.D.) [1963] stated that in early times each planning unit, *khitta*, which accommodated a tribe, had an open space at its centre, called *Rahba*, used partly as a burial space, and partly as a stable for keeping camels and other animals. Although *Rahba* are not systematically present, and though they have no standard form, they were known by local people in each neighbourhood. This is what is known as self-affinity in the absence of a total self-similarity [Batty 2004]. At the *block level*, other common spaces such as dead-end streets, winding alleys, and open common courts called *fina'* were omnipresent. They were used for various purposes, such as playing spaces for children, extended areas for shops, and waiting spaces for allowing loaded animals to pass because of the narrowness of the streets. At the *domestic level*, houses in most Muslim cities were inward-looking, a miniature image of the walled city within its surroundings. Despite the geometric irregularity of the house, the courtyard (*Sahn, Hawsh, Wast ad-dar*) was often square in form and was located at the centre, and thus was a dominant element around which the other spaces evolved.

Self-similarity in traditional housing could also be observed in the growth process at different scales. Old cities were surrounded by an external wall that was successively pushed back each time an extension was needed. Old walls were demolished and replaced by roads giving the city successive ring roads. The new parts then were subjected to an incremental development process similar to that of the whole city. Radial roads connecting the city centre with the city gates ensured the connection of the new extensions to the centre. In Ghardaïa for instance, one can easily recognise the stages of growth by the number of rings, like the technique used to determine the age of trees (fig. 9).

Fig. 9. A general plan of Ghardaïa, Algeria, showing the ring-like pattern that reflects stages of the city growth. From [Didillon and Donnadieu 1977]. Reproduced by kind permission of Éditions Mardaga

One principle of urban management that was applied in most Islamic cities was that harmful activities such as tanneries and blacksmiths were placed outside the city walls. As the city expanded, they were systematically transferred out to the new outskirts. This process was analysed by Raymond [1985] in defining the stages of extension in the old Cairo city.

Growth at the neighbourhood level followed a similar pattern. In describing Fustat, now part of Cairo, al-Suyuti (died in 911H/1509 A.D.) stated that residential quarters were separated by interstitial spaces. When reinforcements arrived each tribe made room for its relatives. Thus quarters got closer to each other and roads were formed [Hathloul 1996: 40].

Similarly, incremental growth manifested itself at a smaller scale in the construction process of plots. Rooms and floors were added each time space was needed and the house was then subject to successive extensions as the family grew in size.

The city could thus be likened to a three-dimensional carpet-like volume that is continuously subdivided into smaller cubes which are in their turn perforated at their centres, a model that recalls those of Sierpinsky [Gleick 1988: 100].

Self-similarity is measured in fractal geometry through a mathematical parameter called "fractal dimension". This parameter theoretically remains the same regardless of how much the object is magnified. It is defined as the power relation between the number of pieces and the reduction factor. $A= 1/(s)D$. Fractal dimension "D" could thus be defined as: $D=\log(a)/\log(1/s)$ [Bovill 1994: 27].

The streams of numbers and calculations behind this equation that are mostly used by mathematicians to identify the dimension of self-similarity through the process of downscaling seem to have no practical significance for architects and planners [Ostwald 2001: 79; Mulligan 1997]. They could instead be replaced by simpler ratios such as open spaces to built areas, and openings (windows, doors) to solid walls. Even the ratio of public sector to private domain could be of interest, although this differs from one city to another. Such ratios could then be interpreted in terms of the distribution of responsibilities between the private and public sectors, responsiveness to climate conditions, land-use pattern, intensity of use, etc. However, a set of maps and plans of different scales from various cities must be used if significant and reliable results are to be achieved.

Randomness and unpredictability

Since its early times, modern science has been influenced by deterministic theories such as Newton's laws of classical mechanics and Laplace's mathematics. Everything in the world was believed to be determined [Cambel 1993:5] by certain conditions. No uncertainty, no chance, no choice, no freedom, and no free will were left in this world; an event was believed to be caused by certain conditions that could not possibly have led to any other outcome. For a dynamic situation this means that, given the initial conditions, the trajectories can be calculated with reasonable precision. Such thought was further confirmed by modern technology and the industrial mass-production system.

In academia, determinism was attractive not only to scientists in physics and mathematics, but to scholars in human sciences as well, such as sociology and economy, who had always struggled to develop accurate means of measuring, achieve exact results and eliminate, as far as possible, anomalies and uncertainties. Architecture and urbanism is a case in point of such an influence. Fine representational instruments, such as sharp Chinese pens and various types of rulers and scales, have always been used to achieve a high precision in plans. The design process in housing, for instance, goes so far as to define the furniture in rooms and offices and the colours of the walls.

However, this approach has shown certain limits in coping with the increasing requirements for changes and variations. Accordingly, new approaches were developed as an antithesis of determinism. Thus randomness and unpredictability have found their way, in theory as well as in practice, to architecture and urbanism.

In traditional settlements randomness and unpredictability have always been the essential ingredients in the development of urban fabric, as most of them were, in modern terms, unplanned and/or spontaneous. Rather, their development was a direct consequence of interactions between people and their environment over the course of decades and centuries. Buildings and urban spaces were subject to continuous changes and transformations that were undertaken by users and/or owners often in the absence of intervention of public authorities and directives of professionals.

For example, randomness can be found in the variety of building types that was generated by the conditions of site, in the varying internal organisations due to family requirements, and in the successive stages of development. This variety is also found in the process of construction which was undertaken by individuals themselves through a system of mutual aid from neighbours and relatives. A similar process also takes place in the open markets as vendors adjust themselves to manage their temporary privatised space according to the rule of "first come, first served" (fig. 10).

Fig. 10. a, above) An open market in Malaysia showing the organic use of space; b, right) An old village in Saudi, showing the organic form of urban fabric made of mud brick

The curdling process developed by Mandelbrot could be of interest in this regard. It consists of producing a "fractal dust" through a randomness generator that can be simply illustrated by a coin-tossing game. A grid of nine squares, which in our case represents a site for housing, is drawn on a piece of paper. Then a coin (or a die, or a random generator) is tossed for each square in the grid, with a defined probability of survival (1/2, 1/3, 2/3). Each of the remaining squares is then subdivided into nine smaller squares and submitted to the same game. Others are discarded. At the third stage, the smaller surviving squares are also submitted to the coin-tossing rule and so on. Ideally this procedure could be repeated to infinity, leaving a dust of points. But for practical purposes, this game could be limited to three or four stages representing different scales of housing. The probability of survival of a square could be varied in order to reflect the real conditions of a site. In other words, the probability of curdling could be increased to 2/3, or decreased to 1/3, according to the potentialities and/or constraints that the site presents. A similar simulation was developed by Akbar [1988: 80-81] in analysing the impact of constraints and opportunities on the development of an urban fabric. Cellular automata software has also been developed to reflect the rules of the game in city development.

In reality, the probability of curdling could be substituted by natural factors (topography, a source of water) and man-made factors (zoning conditions, building regulations) that influence the process of urbanisation and construction. These factors could also be varied according to the scale of urbanisation and the construction process. A further classification into modifying and determining factors, similar to that developed by Rapoport [1969] in his study on cross-sections of housing typology, could also be added in this regard.

The infinite number of development probabilities exemplified in the coin-tossing game could therefore be taken as an image for the magnitude of randomness and unpredictability in the evolution and growth of housing units. It also shows to what extent early conditions could shape urban development of a settlement.

Therefore, in contrast to the deterministic modern approach to housing based on deterministic planning and predictability, chaos theory provides a comprehensive theoretical framework and the instruments for the understanding of the process of the formation and growth of the traditional urban fabric in Muslim cities based on randomness and unpredictability.

Bifurcation

Unpredictability can be best understood through the principle of bifurcation. Sensitivity to early conditions in the coin-tossing game leads to considerable variation in surviving squares at later stages. In other words, the longer the game lasts, the more the cases differ from each other and from the origin in a tree-like form.

A bifurcation diagram is a mathematical tool for representing the development of complex dynamical systems from simplicity to complexity. Computation can be carried out using software to identify the critical points in the curve of changes and different stages before reaching chaos. For example, an initial case “a” of a stream initially splits into two branches, and as water moves further the two branches further bifurcate into four, and then into sixteen, etc. (fig. 11).

In traditional housing, the same phenomenon, but in an even more complex process, occurs. Plots were first subdivided according to the number of users, and were then developed according to each owner’s needs, desires and economic possibilities. Housing units were continuously extended and transformed in response to the changing needs of users. Over centuries, they reached the complexity seen in traditional cities. However, self-similarity was always maintained as a result of the endogenous cultural norms and local, verbally-transmitted know-how. Therefore, in contrast to the same inward-looking housing typology, each house had its own characteristics regarding the position of its components, such as size of plots, entrances, configuration of staircases, courtyard size, number and form of rooms, etc.

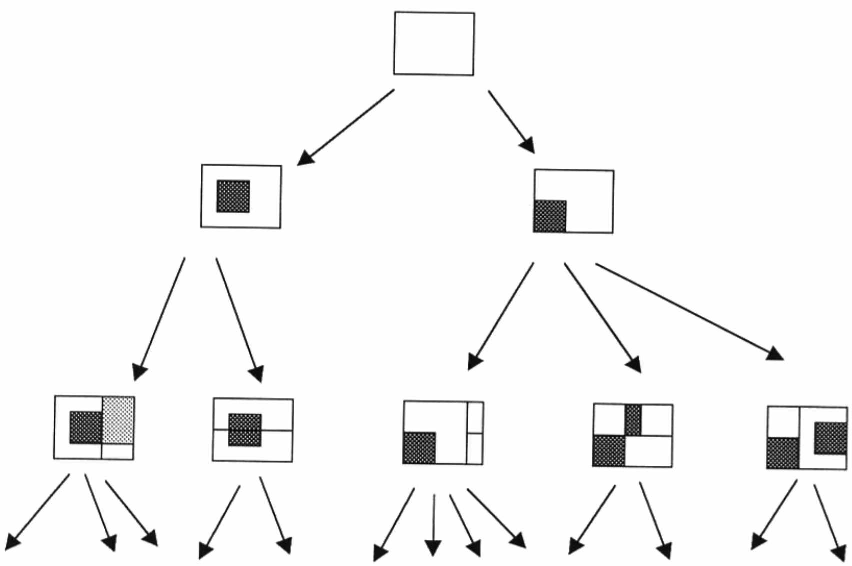


Fig. 11. Possible phases of bifurcation

Bifurcation can also be observed in the form of road networks and their hierarchy. The formation of roads and streets in traditional urban fabric can be considered as the negative space of the built-up areas, since they were defined by the external walls of houses and the process of densification of urban space. The formation of roads was related to the movement of people between places of interest (home, markets, mosques, city gates), and the location of these places combined with the site topography. According to Ribeiro [1997: 294], Y- or T-shaped junctions in the network are of particular interest in describing these movements. "Y" junctions were generally associated with diverging from and/or converging to a place of interest in the primary network of pathways, whereas "T" junctions were associated with the intersection of primary roads and secondary networks of pathways and dead-end streets [Ribeiro 1997: 296]. While the first type denotes a moment of decision as a pedestrian chooses a preferred destination between two equal ones, the second involves a passage to another degree of circulation network that is internal to residential areas exclusively reserved to residents. "T" junctions were therefore often used as gates that were closed during night. Both types were nodes in the bifurcation diagram representing the circulation flow in the traditional urban fabric.

Dynamism, non-linearity and algorithms

Chaos theory is a new paradigm that deals with complex systems. Dynamism is the source of complexity. Dynamic systems can be natural as well as man-made, large and small, regular and irregular in shape [Cambel 1993: 2,7]. Social systems are the most dynamic systems, because they are the most powerful sources of unpredictability, uncertainty and complexity in nature and life.

In the literature of chaos theory, dynamism in complex systems is expressed in terms of causality in different states of a system. According to Young [1994: 11] causality is the probability that, given a set of factors, only that set and no other will produce the same behaviour in the system under consideration. A system in which causal connections can be defined and evolution can be predicted, is therefore called linear, whereas a system in which causal connections are uncertain and only probable is called non-linear.

Complexity in housing stems from its being simultaneously a shelter for the most complex non-linear system – that is, society – and an output of its actions. Houses are made by different hands at different times and placed next to each other in a spontaneous (but conscious) composition. Housing is therefore an outcome of a process rather than a finished product, or as it is sometimes said, it "is a verb and not a noun". Its irregularity, variation, and randomness are the real image of complexity.

Application to housing

A standard rule for controlling complex systems is that the controlling system should be at least as complex as the system being controlled.

One of the widely used technical tools for modelling dynamic systems and expressing linearity and non-linearity is the algorithm. An algorithm is a precisely defined, step-by-step computational procedure that represents causal interconnections in a complex system. In other words, the complexity of a system is most often represented in its algorithm. The longer the string of the algorithmic program, the more complex the system.

There are many sources of complexity in housing. It stems from the multitude of actors (including users, administrators and professionals), the variety of actions (such as new projects or refurbishment, public or private projects), physical characteristics (site, climate, building materials,) and the "rules of the game" described previously.

It is beyond the scope of this paper to elaborate a comprehensive algorithm for housing. Instead, here we further explore the principles of chaos theory in an attempt to understand the urban fabric in traditional Muslim cities. Thus the algorithm technique is applied to the decision-making system in a traditional settlement that is pre-colonial Algiers, which was studied elsewhere [Ben Hamouche 1993, 2003]. The sections below will describe the steps of the two sectors, public and private, in shaping the built environment.

The public decision-making system

According to Batty [2004: 8], cities that are planned entirely or in part are usually of monumental scale, more focused and more regular, reflecting the will of the one, that is, the king or the prince, upon the many. These features also result from the mobilisation and coordination of very large quantities of resources, since this is associated with a central politico-religious power. That can be seen in the following process of decision-making in old Muslim cities:

- Step I: In legal terms, the initiation of public projects and urban development was the prerogative of the ruler. However, the technical formulation of the proposal was often assigned to master builders who were organised into a guild and headed by a trustee who was also an expert in his field. For example, in 1756 the Ottoman ruler of Algiers made a request for the provision of the city with water. On searching for a source of water, an Andalusian master builder named Sta Musa proposed the construction of an aqueduct to catch water from Ain al-Hama spring 19 km away. A similar procedure applies to other public buildings such as the main gates of the city, the city walls, the main mosques and covered markets.

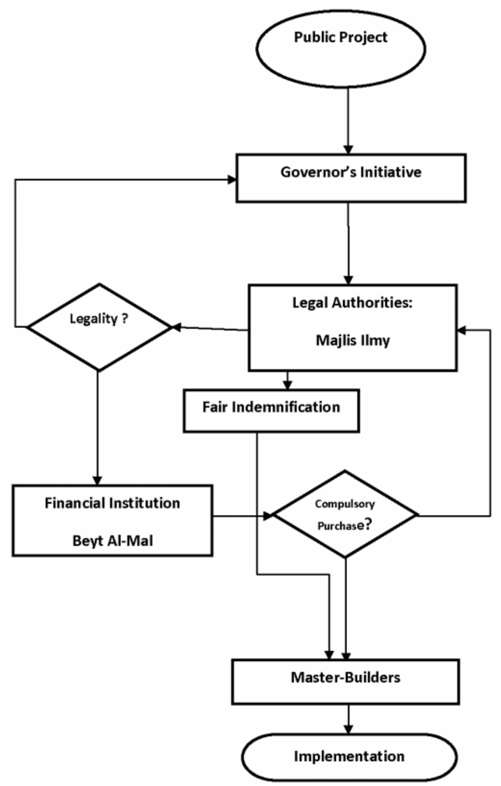
– Step II: The proposal was submitted for approval to the legal authorities. Jurists and judges investigated whether or not the project was legal in the light of Islamic law, which could be summed up in the hierarchic preservation and promotion of the five supreme Ends: Religion, Human Lives, Reason, Human Procreation, and Properties. For example, on examining the fortification of the city and its defensive system, the ruler Pasha Mustafa (1804) wished to build a fortress on a private cemetery because of its strategic location. Despite the sacredness of the cemetery, the legal council decided that defending religion and human lives was more rightful, and thus approved the project.

  However, in less sensitive cases, approval of public projects was subject to the respect of rights and properties of individuals. Owners were indemnified or compensated according to the market price of their properties. The sum was estimated by experts in the field of land marketing and construction.

– Step III: The project was submitted to the treasury for investment. The public treasury, *beyt-al-mal*, was also headed by a judge who was in charge of estimating the fees and the various costs of the project. Expenses were registered in annual registers called *sijillat beyt-al-mal*, in which names of masons, their subordinate workers, and types of work performed were exhaustively listed.

– Step IV: Master-builders were finally appointed for the implementation of the project. Depending on its size and importance, the guild of builders nominated the master-masons to undertake the project. For instance, Algiers was subjected to a serious bombardment by the English officer Lord Exmouth in 1816, and an important part of the city, including the Great Mosque that was adjacent to the coast, was demolished. The guild mobilised all the masons of the city, and appealed to masons from the other nearby cities. The whole population of the city was also involved. According to al-Zahar (1781-1832), reconstruction took one month of round-the-clock work. A century earlier, this kind of construction work was carried out by slaves and prisoners who numbered, according to some estimations, about 25,000 in the city with a total population of 100,000.

The algorithm that represents these steps is linear and comprises only two lozenges from which a "one degree bifurcation" occurs. On the physical level, this is reflected in the Euclidian geometry that characterised the projects of this type. Buildings characterised by simple forms (big domes and arches) and huge scale became permanent landmarks and created an image of the city that lasted for decades and even centuries (fig. 12).



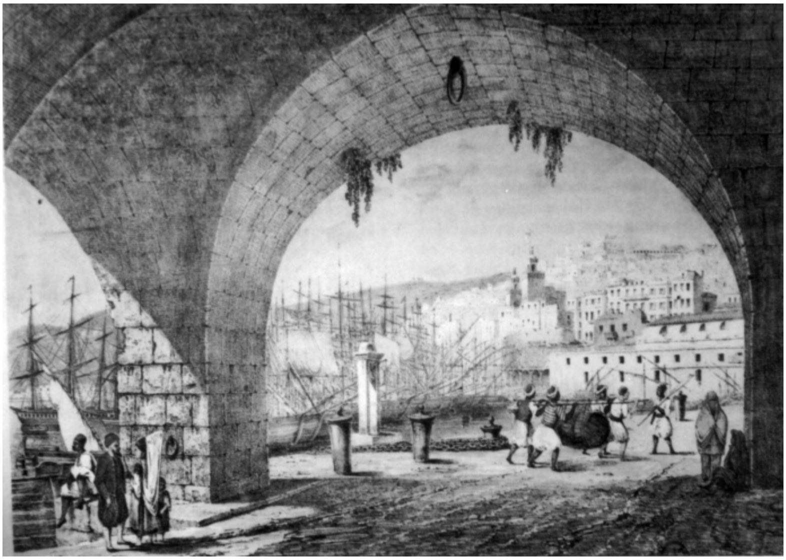


Fig. 12. a, above) the linear model of the decision-making system; b, below) a large arch in lower Casbah, Algiers, constructed during the Ottoman period (1516-1830)

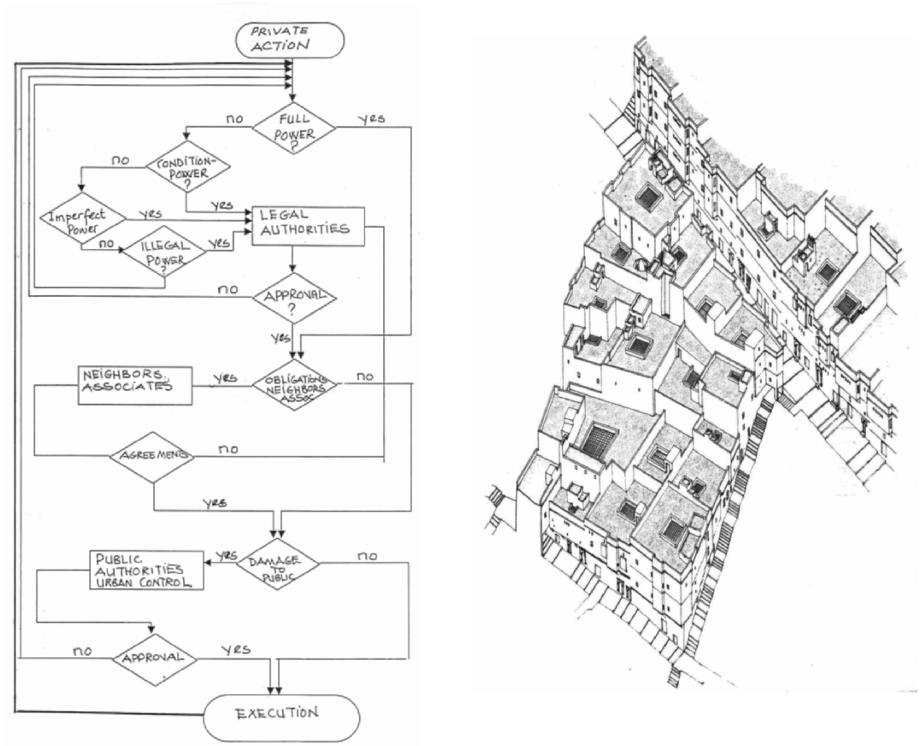


Fig. 13. a, left) the nonlinear model of decision-making system reflecting the individual actions in the built environment; b, right) a portion of the urban fabric in Casbah, Algiers, during the Ottoman period (1516-1830)

The private decision-making system

The incremental process of construction is behind the complex urban fabric. Cities that grow organically developed much more slowly than those which are planned. This natural growth is due to a myriad of individual decisions at a much smaller scale mobilising smaller resources than those of the public sector [Batty 2004: 8]. On the ground, each action was a solution for a set of problems that were dictated by the pre-existing conditions of the site. The algorithm that can be drawn from this process could be organised into many "loops" that represent the principles governing the private actions previously discussed (fig. 13).

– Step I: Ascertainment of the freedom to act. An action could not take place unless the person acting was legally qualified by having the full legal power, *Wilaya tamma*. This was the case of most residents who were mature and acted on their own properties. There were three exceptions: persons with conditional power, such as the trustees acting on behalf of charitable institutions; those with partial power, such as children who did, however, own assets; and people with illegal power, such as squatters. A permission from a legal authority for any action on properties in the hands of the person acting was requested.

- Step II: The action was examined in relation to its immediate surrounding. It could be carried out if no prejudice was caused either to neighbours and to the public, depending on the nature and location of the property under consideration. Regarding the public interest, private properties which were a source of harm, such as those generating smoke or odours, or those obstructing circulation, were generally blocked by the direct intervention of public authorities, the *Hisba*. For example, in Tunis, the judge of the city Abderrafi'(1300 A.D.) was once asked about public baths (and other similar activities) causing smoke and/or odours. He replied that unless owners managed to cope with the harm, they should be blocked [al-Rami 1995: 320].

- Step III: Objections of neighbours and associates to a private action were in most cases overcome through negotiations and agreements which led to exchanges or purchase of rights and interests, or to a friendly relinquishment. For example, a resident could purchase or rent a right for the drainage of his rain-water or sewage on his neighbour's property, or obtain it free of charge, by way of charity.

- Step IV: Pursuance of the project. In spite of the acting person's failure to establish agreements or to get relinquishment from his neighbours or associates, he nevertheless persisted in carrying out his project. At this stage such cases were taken to court. Depending on the verdict, the person acting was then either allowed to proceed or was blocked.

Compared to that of public actions, the algorithm of private actions comprises many loops which denote the spectrum of alternatives, and thus the complexity of the system. In other words, the algorithm of private actions is a non-linear system and is characterised by a high degree of bifurcation. On the physical level, this is reflected in the fractal geometry of private buildings and residential areas.

This process took place each time a building action was undertaken, however small, whether opening a window, relocating a door, or adding an upper room. Despite the limited number of "rules of the game", the final result at the urban scale over a given period of time is unpredictable, just like the coin-tossing game of squares described earlier.

This algorithm also expresses the mechanisms of self-organisation and self-renewal in traditional settlements. In most cases, private actions took place without any intervention by public authorities. Agreements, easements and/or relinquishments among neighbours and associates were established amicably and in presence of other neighbours as witnesses. Only the major cases of dispute were taken to court; these were few in number compared to the continuous flow of actions and consents. In other words, this system was based on the social balance that comes out of the conflicts of interest which urban life continuously breeds, and which generate complexity.

Conclusion

Chaos and fractals is a relatively new theory for studying complexity. It provides concepts and tools that can also be used in analysing the hidden order underlying the irregular geometry of traditional urban fabrics, which was long considered to show no order at all.

Far from comprising a mere increase in size and an enlargement of scale, the internal structure of traditional urban fabrics was characterised from the domestic level to the city level by a striking degree of self-similarity. Randomness and unpredictability explain the variety of forms and types of houses that reflected the problem-solving process of construction and the influence of early conditions of site on the overall form of the city. The technique of bifurcation explains how the houses diverged from a simple piece of land and an inward-looking common model to an infinite number of house types modelled by the variety of persons acting and their needs, and specific conditions of the site.

One way to grasp this complexity and dynamism is by establishing an algorithm for the process of users' actions that shaped the urban fabric. Two systems could be identified in this regard; the linear system and the non-linear system, representing respectively public action on one hand, and private and collective actions on the other, each of which generated a particular geometry, mostly Euclidean in the first case and fractal in the second. However, the two types interacted and came together to constitute the balanced urban fabric of traditional cities that we see today.

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Mustapha Ben Hamouche holds a Doctorate from the Institut Français d'Urbanisme, Paris VIII University; a Master of Philosophy (M.Phil) from the Faculty of Arts, University of Newcastle upon Tyne and a Diplome d'Etat d'Architecte (BSC) from EPAU, the Ecole Polytechnique d'Architecture et d'Urbanisme, Algiers. He joined the University of Blida, Algeria in 1986 and taught there for nearly ten years. He was also in charge of postgraduate studies and research programmes there. From 1995 to 1999 he left academia and worked as an Expert Planner in the Town Planning Department of Al-Ain, UAE. He is now Associate Professor at the University of Bahrain. He has published books and papers in Arabic, English and French on urbanization in the Gulf, urban history in North Africa, and the impact of Islamic law on the built environment in traditional Muslim cities. At present he is working on the application of chaos and fractals theory to traditional urban fabric. He is also leading a research team at the University of Bahrain on the application of GIS to Urban Development in Bahrain.

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Research

Antonelli's Dome for San Gaudenzio: Geometry and Statics

Abstract. In this brief note the authors describe the studies undertaken to date regarding the dome of the Basilica of San Gaudenzio in Novara designed by Alessandro Antonelli, with particular emphasis on the geometry and statics of the external dome. Following a brief summary of the events and vicissitudes attendant on the construction of the dome, the structure will be examined from the point of view of the geometry, the construction techniques, and the materials used, in order to clarify the static behaviour and the stability of the whole set of structural and constructive elements of which the dome is composed. These studies have allowed us to obtain the necessary information for evaluating the complex and ambitious structural achievement of a significant element of Antonelli's basilica.

The historical background

In order to introduce the reader to an in-depth understanding of the object of study presented here, we think it is important to precede the geometric and structural considerations with some brief notes on the history of the construction of the cupola of San Gaudenzio, built to a design by architect Alessandro Antonelli (1798-1888) between 1841 and 1878 [Rosso 1989].

The Basilica of San Gaudenzio was built on the highest point in Novara between 1577 and 1690, its original construction entrusted to Pellegrino Tibaldi, following instructions formulated by the Capitolo della Fabbrica and the ecclesiastic community [Peagno 1998a]. The construction of the monumental dome began much later, in 1841, and continued until 1878. Work on the dome was accompanied by constant worries regarding the solidity and stability of the structure, giving rise to numerous interruptions, diatribes, investigations and inquiries aimed at evaluating whether or not Antonelli's project could actually be built. The construction went through four successive designs by Antonelli, who, taking advantage of a suspension of the work between the years 1851 and 1860, made significant formal and constructive revisions to his own initial project [Peagno 1998b]. In fact, Antonelli intended to construct a bold and majestic work, even though he knew how difficult it would be to obtain approval of a project so highly complex in terms of construction technology and statics, no matter how enlightened the committee was. In fact, his first project did not yet include the formidable arches that would be effectively constructed to support the dome.

In 1863, construction had risen to the level of the thin-shell exterior dome, supported by a external double ring of granite columns (fig. 1). After yet another, prolonged suspension of the work, in 1876, the small lantern at the top was finally completed in 1878. The statue of San Gaudenzio was placed at a height of some 125 m above the pavement of the church.

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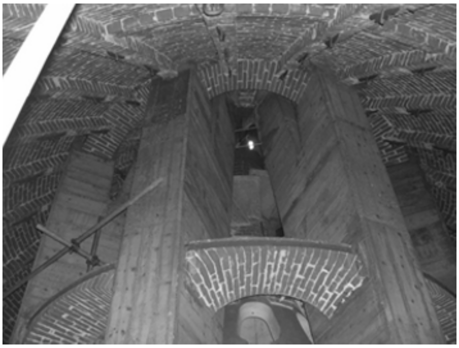

Fig. 1 (left). View of the double order of Corinthian columns of the dome and lantern

Fig. 2 (above). Additions in reinforced concrete built by Danusso in the 1930s

In 1882, following the formation of some cracks in the flat arches that form the supporting skeleton of the drum of the cupola and the lower arches, the four piers that support the dome – part of Tibaldi's original work – were reinforced, or better, completely reconstructed, using a material and construction technique that were more reliable and in keeping with their intended function than the original ones.

In 1929, the observation of some cracks in the capitals of the first order of the lantern led engineer Arturo Danusso to alleviate the problem by applying a covering of reinforced concrete to the lantern and to the pilasters of the conic structure below it (fig. 2). This was complemented by the partial filling with concrete of the voids inside the piers that support the dome, with the aim of stabilizing the dome itself – also in danger of collapse – but at the cost of a considerable increase of weight [Pozzi 1997]. In any event, the new structure in reinforced concrete, when combined with the original one in brick, completely altered the original statical system, and in fact led to an infinite series of problems relating to the statics and the stability of the entire basilica, and raised questions about the feasibility of Antonelli's design that persist to the present day.

Following the interventions proposed and executed by Danusso and the persistence of phenomena related to the formation of widespread cracking, the basilica as a whole has been subjected to long-term and on-going monitoring with the aim of demonstrating the structural instability of Antonelli's project – an instability which, in fact, does not exist [Corradi 2000].

Architecture and structure

The dome of the Basilica of San Gaudenzio is noteworthy above all because of its monumental scale, with a height from the floor level of the church to its top of 125 m, an internal diameter of 14 m, and an external diameter of 22 m, as well as for the daring of the forms and the complex constructive system, all of which make it unique in European architecture of the nineteenth century. It is also the highest masonry building in Italy.

Specifically, the study presented here concerned the exterior dome, that is, the portion of the structure that rises from a height of 65 m, corresponding to the second order of exterior granite columns located at its base, up to 85 m, the level of the base of the lantern. It is necessary to distinguish the interior (lower) dome from the exterior (upper) dome. The former is visible from the interior of the Basilica and is decorated with plaster coffers; the drum under it was encircled by a ring of reinforced concrete in the 1930s, part of the interventions of consolidation realized by Professor Danusso, with the aim of avoiding the "ovalisation" and implosion of the dome (fig. 3). The uselessness of this intervention has been recently demonstrated [Corradi and Filemio 2004].

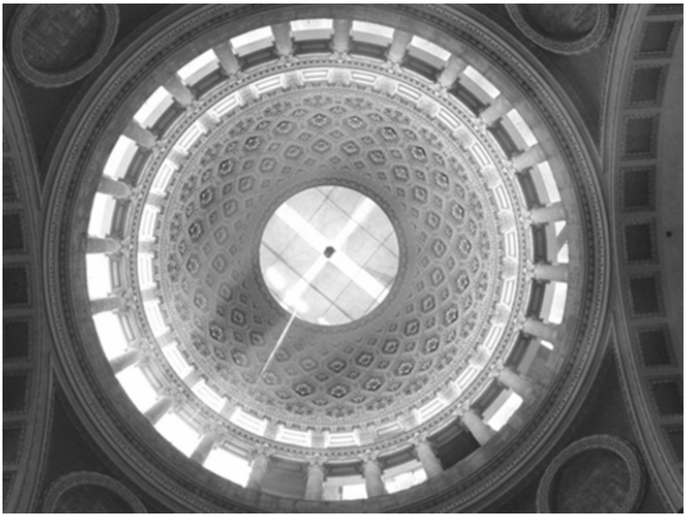

Fig. 3. View of the inner dome, the so-called "*gran tazza*", decorated with floral motifs and coffers, and of the ring in reinforced concrete built by Danusso at the level of the drum

The construction is daring and ambitious, especially since Antonelli designed and built a dome of such enormous scale with a thin structural shell only about 12 cm thick (the width a brick header) stiffened by ribs and compression rings – all in brick – arranged in a system of "meridians" and "parallels". This surface, stiffened in its turn both at the base by the ring of the drum and at the top by another ring in brick, conceals within itself another spectacular architecture, that is, a stiffening structure shaped like a truncated cone composed of a system of inclined brick pilasters placed on juxtaposed planes forming a grid with a circular base that decreases in diameter as it rises, joined to each other by a system of relieving arches (figs. 4 and 5). This structure constitutes the actual loadbearing skeleton designed by Antonelli, the element that is key for the statics and stability of the entire building.

Although the external dome and the internal structure are interconnected, they have in fact their own independent static behaviour, almost entirely consequent to different static stresses due to external forces and to the loads carried. These behaviours, however, are similar since each transmits its dead load through pre-established points and lines of force: the external dome by means of the ribs that subdivide it into wedges, the internal structure by means of the grid of pilasters and ribs of the truncated cone. On the other hand, the exterior dome, supported by the mighty arches at the base, resting in their turn on the large piers in the crossing of the nave, is in fact only a covering for the internal cone. The entire composition, combined with the lightness of the structural system and the inspired geometrical control of shape and light, testifies to a level of ambition and acumen found in few other buildings of the same time period [Benvenuto 1987].

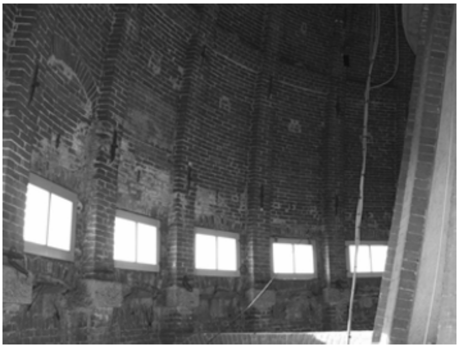

Fig. 4 (above). The system of internal and external ribs with granite dados, with the insertion of relieving arches and flat arches

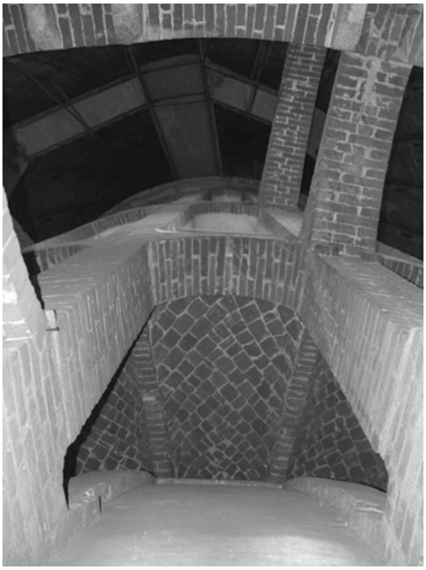

Fig. 5 (right). The junction of the interior cone seen from below

Our studies of the geometry have focused above all on the principal elements of the thin-shell dome, that is, the larger arch (the meridian) and the vault (the thin shell between two meridians). The meridians are stiffened by the complex system of rings along the parallels, and constitute the principal skeleton of the structure, while the vault is a thin shell, impalpable, almost ethereal. From a first examination of the structure the vault appears superfluous in terms of the statics in the stability of the whole but in reality it plays two distinct roles. The first is that of connecting the system of ribs like a membrane, following the meridians and parallels; the second is that of supporting the external covering, made up of slabs of schistose stone. In particular, the object of the present research was to identify by means of an analysis of different typologies of rotated surfaces (those belonging to families of conics such as the parabola, the catenary, etc.) which one effectively describes what happens in the dome. After a careful analysis of the dimensions, backed up an in-depth instrumental survey of the geometry of the dome, we found that Antonelli relied on the circular arch to govern the execution of this magnificent architectonic object, a hypothesis validated by the existence of the compass actually used by Antonelli to trace the curve of the dome and to design the profile of the centering for the worksite. Still functioning today, the compass is housed in the so-called *Sala del Compasso*, located above the east transept of the basilica.

Further, noting that the curve of the dome in section is very acute, its height much greater than its width, our studies have aimed at revealing some sort of analogy with the methods used to lay out the arches and vaults of Gothic architecture, finding, if not a perfect analogy, at least a resemblance to the pointed lanciform arch. The profile of the lanciform arch is characterized by the fact that the generating radii of circular arches of the dome are located at the level of the impost of the dome and have a diameter that exceeds the height of the object laid out, with a ratio equal to 1:0.80. We have found that the ratio of the diameter to height of the dome of San Gaudenzio is equal to 1:0.75. This leads to a displacement towards the exterior of the section of the centre of the layout and a curve that is even more acute than that of the Gothic.

The dome is thus a geometric form obtained by the rotation of a curve along the circumference of the base about its central axis, which is also the centre of gravity for the system of weights. It is therefore a rotated solid characterized by the perfect radial symmetry of the structure. Further, as surveys and studies undertaken on site have shown, the dome is not subject to phenomena of "ovalisation", since even though its diameter decreases as its height increases, it still remains a geometric figure characterized by perfect circularity at every horizontal plane, with infinite axes of symmetry and constant internal diameters.

Construction technology

The dome that covers the truncated cone structure is made up entirely of a single course of solid bricks and mortar forming a very thin shell (about 12 cm thick). The dome is supported by four double arches of different heights that spring from the same impost level but which are developed on different vertical planes and with variable slopes (oblique arches). The keystones, the imposts, and points just above the imposts are marked by the insertion between the bricks of stone blocks that correspond to the nodes of distribution of the stresses.

Fig. 6. Oblique arches that support the dome showing the stone blocks that stiffen them.

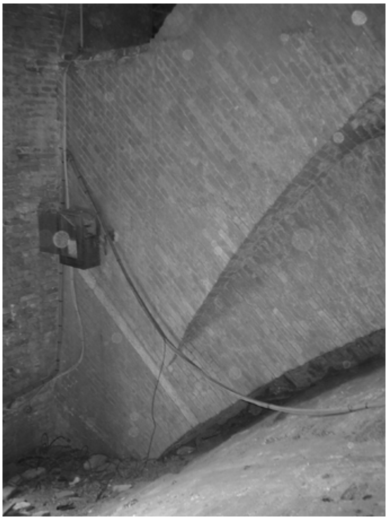

Fig. 7. Detail at the intersection of the two oblique planes of the arches

Further, in some courses the bricks are cut into wedge shapes and specially formed to follow the curve of the individual arches in order to guarantee the most suitable mechanical and material behaviour. On the exterior the dome rests on a double ring of monolithic Corinthian granite columns, which following Danusso's interventions lost their original static function of transmitting to the ground, like Gothic buttresses, the loads of the structure (figs. 6, 7).

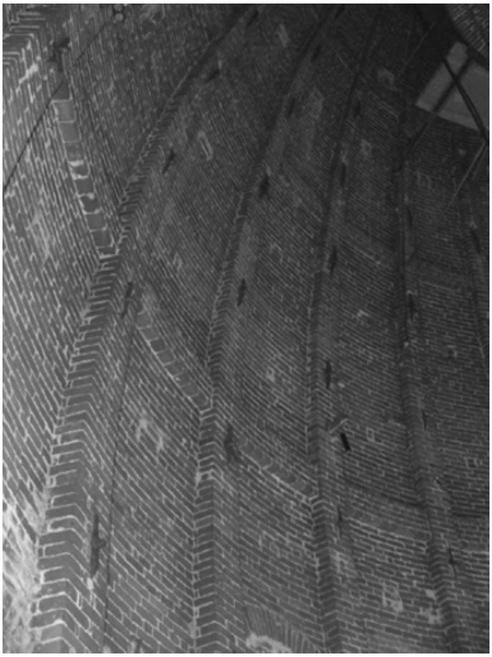

Fig. 8. A view from below of the external shell in brick stiffened by the system of ribs following "meridians" and "parallels". The dome, a shell only one brick header thick, is stiffened by a system of principal ribs (meridians) about a header and a half thick and of secondary ribs (parallels) one header thick

The skeleton of the dome is composed of thin ribs, or meridians (fig. 8) which, beginning from the impost of the dome (that is, from the top of the drum), rise up to the ring at the top of the thin shell. This upper ring, together with the lower ring at the drum, confers stability and stiffness to the exterior vaulting that covers the masonry skeleton of the ribs. The ribs rest on granite dados placed atop brick pilasters. The dados mark the transition from the vertical to the curved elements and ensure that the stresses that are transferred from the top of the dome to the impost of the pilasters and from there to the drum do not generate states of eccentric compression-bending stress outside the central core of inertia of the masonry section. In this way, the section, even though slightly in compression-bending, is subject only to forces of compression.

The ribs are connected to each other by a system of "chains", these also in brick, placed on three levels ("parallels") which give the dome the stiffness required to remain in a situation of stable equilibrium in the event of possible settling or failures due to horizontal forces such as wind or earthquakes.

These in fact translate into a series of compression rings that guarantee the stability of the thin shell, given its height and the thinness of the masonry. It should be further noted that the structure has been made even lighter by the opening of two series of circular and rectangular windows all along the perimeter of the dome, a feature for which Antonelli has often been criticized because it seemed to further weaken the masonry. In reality, the static equilibrium is maintained thanks to a continuous system of flat arches and relieving arches between the openings to guarantee the stability and stiffness of the masonry fabric.

At the impost of the exterior dome – as we said, at 65 m from the ground – is placed the 1 m wide catwalk of the dome, that is, a walkway in brick masonry that develops circularly along the vaults, supported at the intrados by a series of arches resting on granite corbels (fig. 9). This walkway functions as an internal chain, helping to ensure that the dome does not deform into an oval, thanks to its high degree of resistance to bending and to the complex static mechanism of which it is made. Further from this catwalk take off a ring of upside-down arches that connect it to the first dome on which rest the pilasters of the truncated cone.

Fig. 9. Catwalk of the dome supported by granite corbels connected to the relieving arches

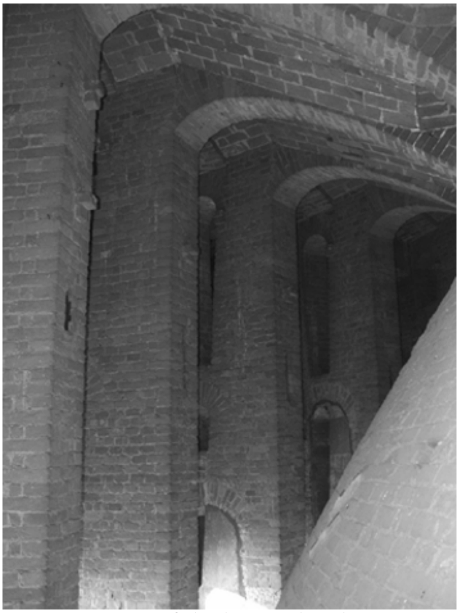

Fig. 10. system of climbing arches that support the catwalk above the interior dome

Fig. 11. View of the upside-down stiffening arches

Fig.12. Ribs and supporting system of the external covering in stone slabs

Fig.13. External covering: dados and restraining rings of the finishing in correspondence with the internal rib system

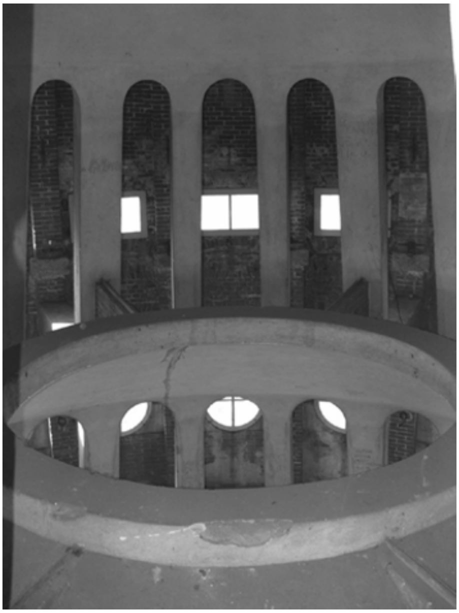

Fig. 14. Detail of Antonelli's "mechanism" under the exterior dome

The truncated cone structure, today entirely covered with cement, is composed of the system of pilasters forming the connection between three circular domes whose diameters decrease as they rise in height. The pilasters are connected and stiffened by brick arches and converge at the circular ring at the top of the dome, together with the ribs. That structure is connected to the dome by means of relieving arches in brick characterized in correspondence to the first dome, by a flat extrados and an intrados of flattened arches, which connect the individual pilasters to the corresponding ribs. Along the entire height of the cone is found a complex system of upside-down arches that connected to the outer dome (figs. 10, 11).

As can be seen from this brief description, this is a complex masonry "mechanism" capable of resisting the set of horizontal and vertical forces – static and dynamic – to which the structure of the basilica is subject (figs. 12, 13, 14).

Static behaviour

The investigation of static behaviour and the stability of the dome focused on some of the principal aspects and particular elements of the structure, such as the arches of the interior dome and the external shell, visible from the interior of the vaults between the ribs, first taking them into consideration as individual structural elements and then as interconnected elements, evaluating within their sections the state of stress that results from the force of the loads and external forces of a static nature (that is, we have also considered the force of the wind and earthquakes).

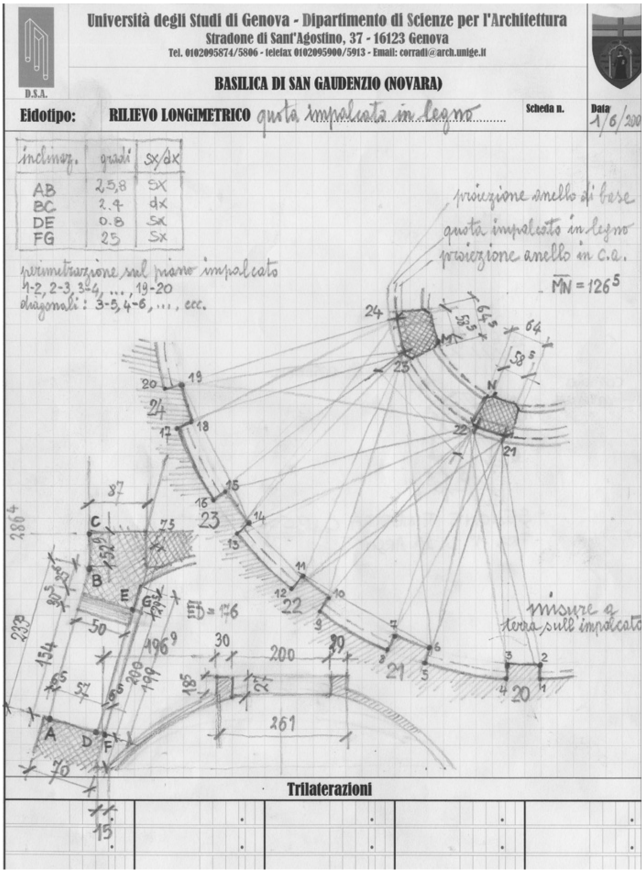

Fig. 15. Geometric triangulation (eidotype)

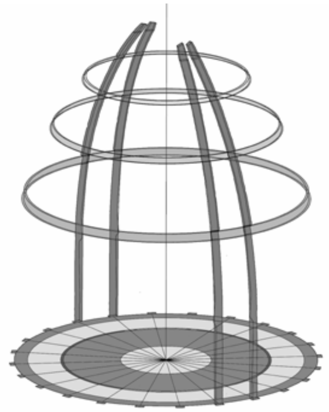

Fig.16. Schematic depiction of two symmetrical ribbed segments

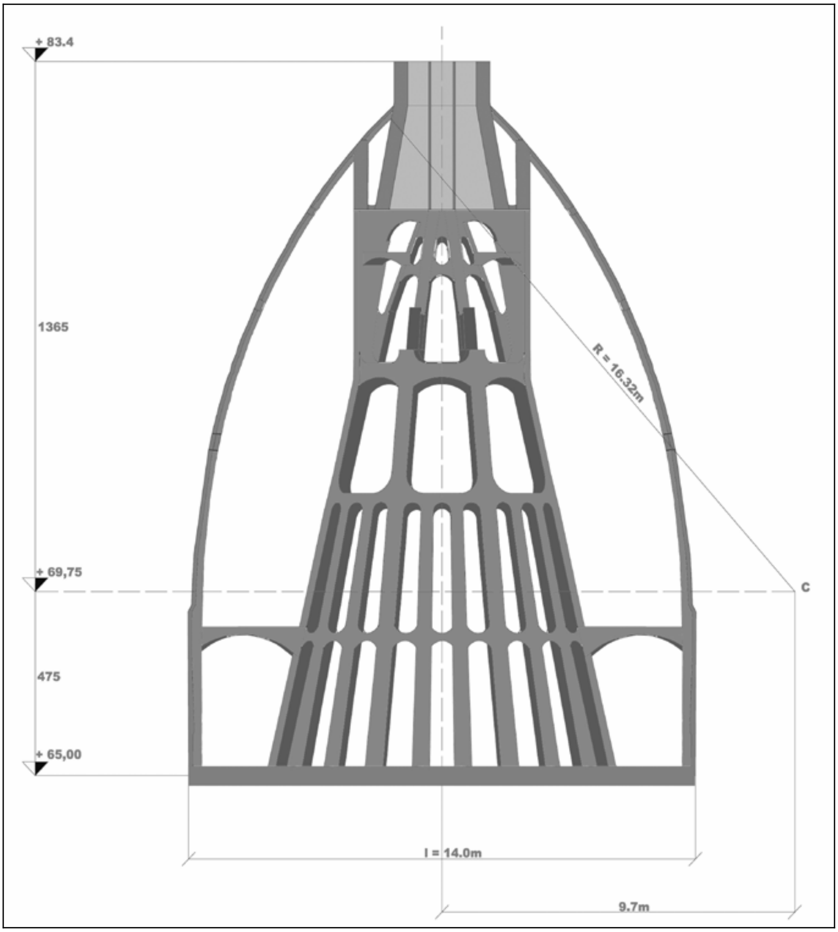

Fig. 17. Schematic depiction of the generating radius of the exterior dome

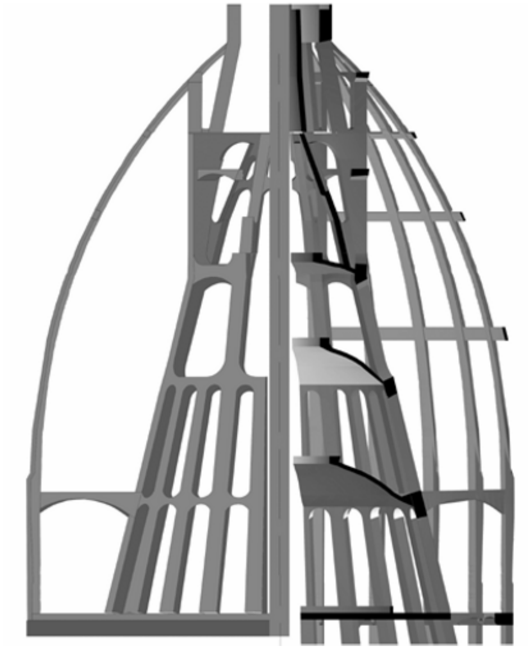

Fig.18. The section of the exterior dome and the interior cone

The modus operandi of our investigation was that of extrapolating the sections of the two structural elements by means of an accurate survey and an in-depth study of the geometry (figs. 15, 16) to determine the following: centres of gravity, the relative weights of individual blocks comprising the arch, the vaults, and the external shell (figs. 17, 18); the centres of stress; the kinematics of the thin shell as a whole stiffened by the ribs, and by the elements that connect the central cone as a function of the various kinds of stresses. The force diagram (by means of catenary and deformed elastic curves with limited rigid-plastic behaviour) and the resulting verification of the line of thrust curves and allowable deformations, useful for understanding the mechanical behaviour of the structure when subjected to various external forces, and, above all, the study of the states of tension within the resisting sections by means of the finite element method, with elements of eight nodes to identify the bricks and mortars, have demonstrated that the materials used in the construction are in fact adequate to resist the stresses.

The various investigations conducted on the structure of San Gaudenzio have shown that the shape of the line of thrust is perfectly contained within the masonry section of the dome, with only minimal variations – due to external forces, particularly the wind – from its intrados to its extrados, respectively from the keystone to the impost and vice versa. Further, a careful examination of the states of compound stresses has shed light on how, in particular load conditions (dead load), the behaviour of the dome as a thin shell, that is, capable of resisting bending, comes very close to the behaviour of a membrane. In fact, the shape of the arch of the median axis becomes a very similar to that of a segment of a catenary, with the geometric difference of an average quadratic deviation well below 1%.

The statical and mechanical analyses performed on the structural system of the dome as a whole have shown that up to the third parallel the structure is in fact only subject to compression, while below this level values of zero tension are measured at the intrados, with small values of tensile stress due almost entirely to numeric approximation. In any case, the values corresponding to maximum tension and minimum compression and tensile stress fall within the admissible range of resistance relative the materials used in the construction, that is, solid brick and mortar (fig. 19).

Finally, the study of the collapse kinematics are reminiscent of Poleni's notes regarding the stability of the dome of St. Peter's by Michelangelo.

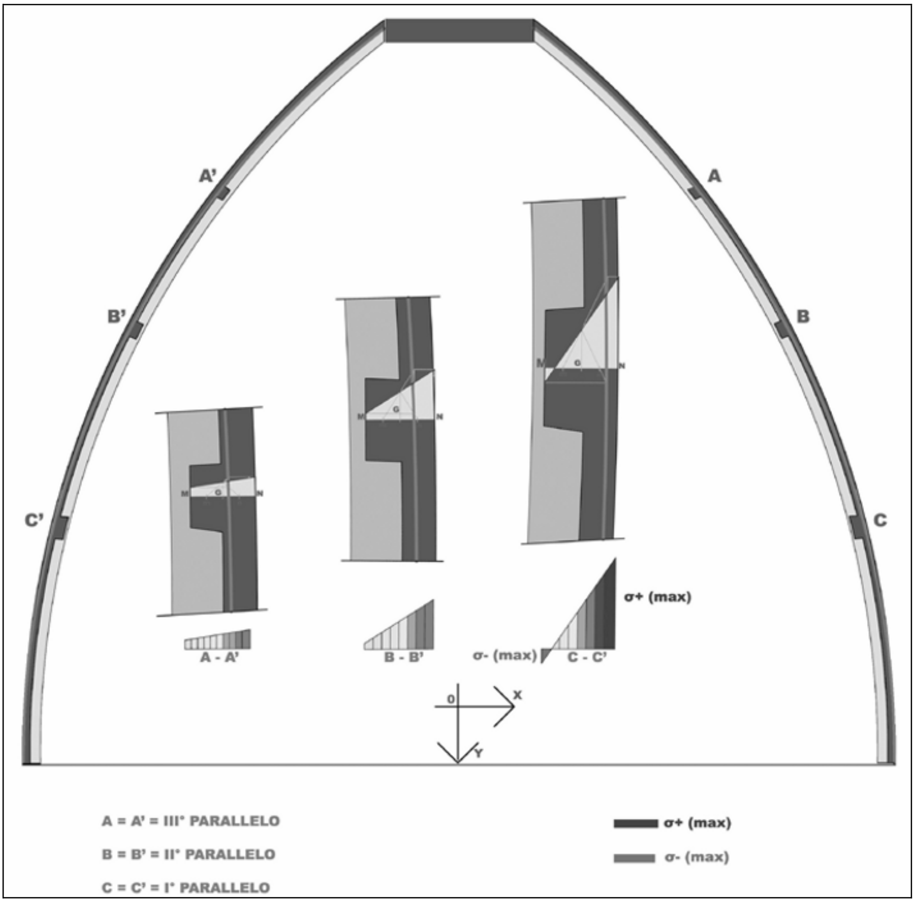

Fig. 19. Determination of the state of the principal stresses

Conclusions

The results obtained from the studies of the geometry and the mechanics of the dome of San Gaudenzio demonstrate how exceptional the structural system design by Antonelli is (fig. 20). It also demonstrates how such a grand and magnificent object can be in fact slender and light. The system used to stiffen the dome (comprised of the system of ribs arranged along meridians and parallels, which connects the drum to the ring at the top) allows it to adapt to possible small variations of the initial state of equilibrium caused by failure and/or settling of the underlying structure or by external forces (wind, earthquakes) without incurring the risk of causing collapse kinematics. Further, if the brick dome seems astonishingly light, we should recall how Antonelli's masonry system, even though its main task is loadbearing, makes the various structural components evident and clearly distinguishes the principal from the secondary sustaining elements, relying on the primary system of vertical elements and the central cone for stability. Order and equilibrium govern and harmonize all the elements of the building, while a set of stiffening elements, embedded within the masonry itself, guarantees the invariability of the statical-mechanical system.

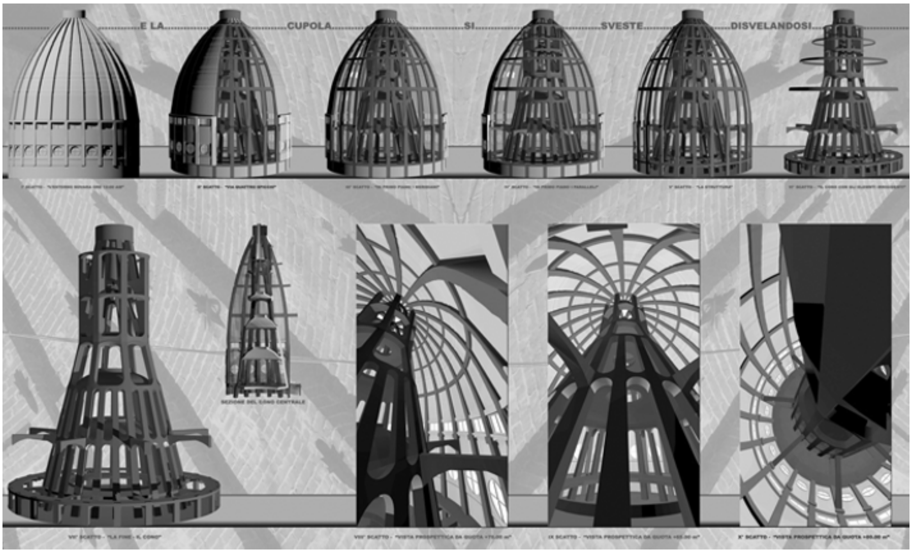

Fig. 20. Antonelli's "mechanism"

Translated from the Italian by Kim Williams

Acknowledgments

The present study was conducted by the authors with the active collaboration of Massimo Trenetti who, as part of the preparation of his master's thesis in architecture, participated in the survey campaigns and the analyses in situ and was responsible for the drawings relative to the geometry of Antonelli's structure and the CAD drawings preparatory to the analysis of the statics. We also wish to thank Arch. Paola Pesce and Dr. Federica Lercari, specialists in the restoration of historic monuments, for their valuable and intelligent collaboration during the phases of the survey and the analysis of materials, Arch. Guido Peagno and Prof. Paolo Piantanida of the Politecnico di Torino, and all of the directors of the Fabbrica Lapidea of San Gaudenzio for their cordiality and helpfulness during the various phases of the work and for ongoing and helpful exchange of information on the history and constructive techniques of Antonelli's building. Finally we thank Kim Williams for the translation into English of the original Italian text.

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Keywords: music, architecture, Daniel Libeskind, Bela Bartòk, Steven Holl, Peter Cook, Ernest Bloch, golden section

Research

Music and Architecture: A Cross between Inspiration and Method

Abstract. This paper is one of a set of lessons prepared for the course of "Theory of Architecture" (Faculty of Architecture – "La Sapienza" University of Rome). The didactic aim was to present – to students attending the first year of courses – some methods for the beginning stages of design and their applicability to any kind creative work. The brief multimedia hypertext quoted at the end of this paper was carried out in collaboration with the "LaMA" (Laboratorio Multimediale di Architettura) as a test for new educational tools applied to first our "e-learning" experiences.

Introduction

The analogies, coincidences, affinities and bonds existing between architectural and musical compositions have been the object of research since ancient times. Traveling through the history of this theme is very interesting, especially when it is possible to identify the social and cultural aspects that are interpreted in the different forms of composition: pictorial, poetic, musical and architectural. In this regard, for those who are interested in the study of the ways in which the contemporary architect works, one question appears central: How do projects (often very well known and in some way part of the collective cultural memory) that are explicitly declared to derive from musical pieces, pursue that intent? The academic approach seems to fluctuate between scientific operative methods and an aesthetic method, where a subdivision between the practical and theoretic spheres is still acceptable.

A study structured in this way is part of a larger reflection on the critical reading of an architectural project aimed at decoding the graphic signs and their motivations, expressed or implicit, conscious or unconscious, which are part of a group of symbolic memories and of intermittent recollections of the most powerful icons of our academic formation. The same symbols also belong to other fields of knowledge, and the capacity to de-codify them globally is part of the architect's long formative journey. Hypothetically, this journal is never-ending, in that it does not simply constitute a basis of indispensable knowledge for the architect in training, but becomes an essential part of the development of each individual architect's personal way of working, and of how abstract ideas are translated into the concrete spaces of everyday life.

The goal of this present paper is to study three works of architecture that explicitly refer to works of classical musical rather than to vaguer generic principles of harmony and musicality, for which a serious comparative analysis becomes more difficult. Each of the three works takes a different approach to its particular musical theme.

Music as Inspiration

Daniel Libeskind's analytical work operates in the field of architecture's second invariant, defined by Bruno Zevi as the study of asymmetry and dissonance that is realized

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in the conscious application of a design method which results, on one hand, from the illogical chains produced by liberal associations of the mind and, on the other, from the logic of the deformation as a singular case of the variation of the composed theme, of the topological order, of the deconstruction. A direct consequences of this is what is defined as a rarefaction or dissolution of the architectural sign, which in reality leads to an abstraction that is often extreme, but also to a closure that architecture shares with all other forms of artistic expression.

The consequential and inevitable reduction to silence in "compositional writing" has been compared to other, more recent musical forms where the paroxysmal crescendo of the sound are contrasted by sudden long pauses, both metaphoric expressions of the contemporary condition. These concepts may be expressed in a slightly cryptic way in a collection of drawings that Libeskind entitled "Chamber Works",[1] in the same cryptic way that some contemporary scores adopt a system of notes without the staff.

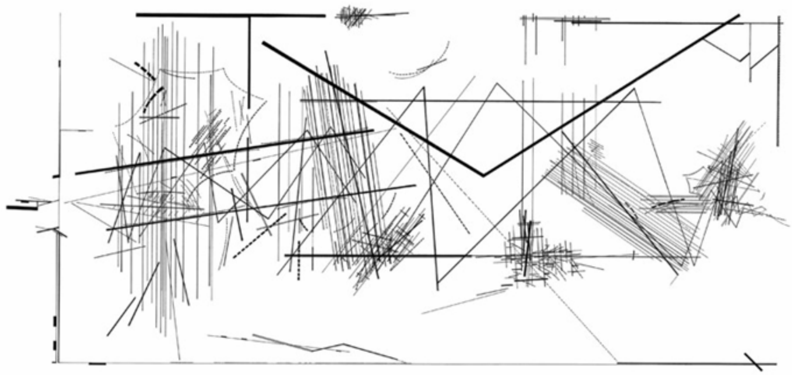

Fig. 1. Chamber Works, drawing by Daniel Libeskind [1983]. Image courtesy of Daniel Libeskind

The title "Chamber Works" in itself evokes a "chamber architecture" in the same way in which we might speak of "chamber music", a complete composition in all its parts, realized through the use of a reduced number of elements, only those absolutely necessary to give body to the logic of the written text. The two series of these drawings, the horizontal and the vertical, form a continuum of graphic inventions that Kurt W. Foster [Libeskind 1991] defines as "spatial music", a kaleidoscopic collection of lines and symbols that represent the same double axial structure of sounds; melody and/or chords, horizontal and/or vertical structure, regulated by the common principle of liberal variation (fig. 1).

This methodological process, experimented in the pictorial form in "Chamber Works", is also applied in the project of the extension of the Berlin Museum with the section dedicated to the Jewish Museum Department [Libeskind 1992], where even in a constructed architecture the permanence of a design idea derived from the chance vicinity of apparently heterogeneous graphical points, is realized.

The topological deformation of the six-pointed star in the plan of the Berlin museum, the figure that generates the idea of the place, is the Star of David, transformed from a neutral symbol of religious faith into the memory of the holocaust through the alteration of the traditional geometry. This star, disjointed and no longer recognizable, becomes the path through the museum (figs. 2, 3, 4, 5).

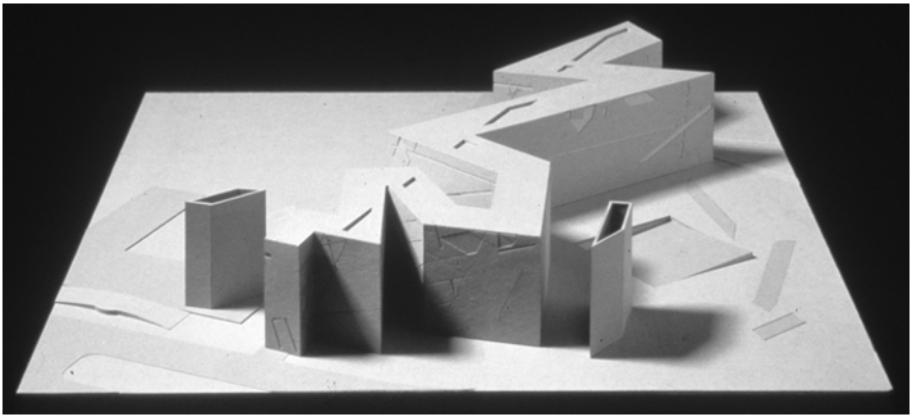

Fig. 2. Aerial model of the extension to the Berlin Museum with the Jewish Museum Department. ©SDL. Image courtesy of Studio Daniel Libeskind

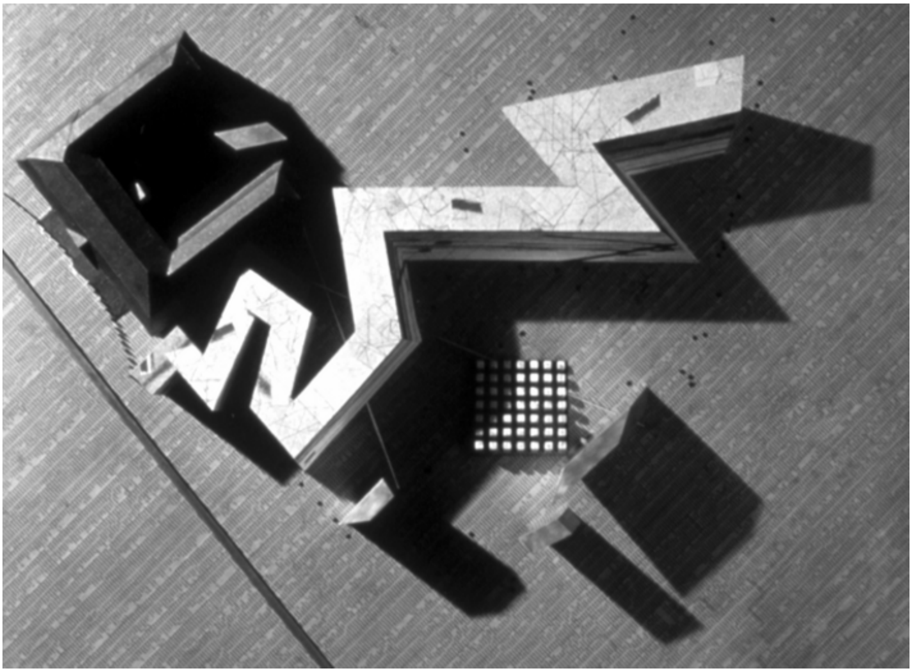

Fig. 3. Plan view of the model of the extension to the Berlin Museum with the Jewish Museum Department. ©SDL. Image courtesy of Studio Daniel Libeskind

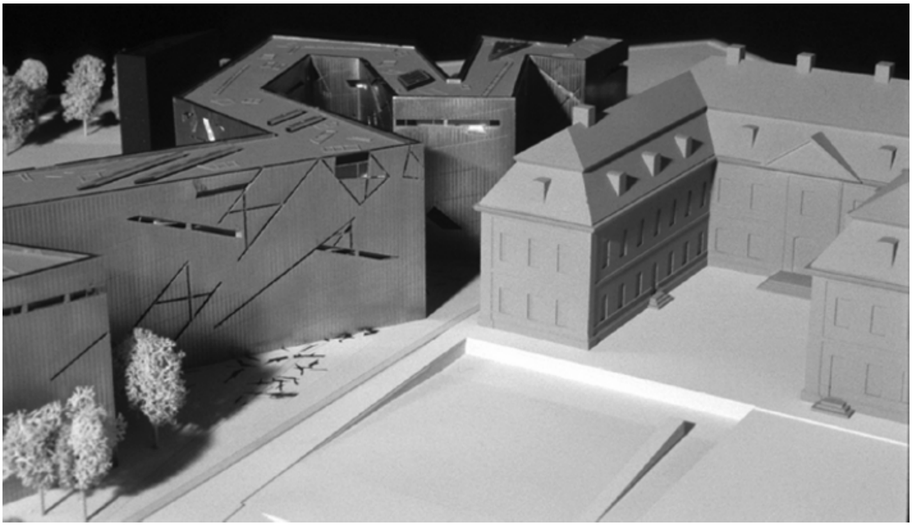

Fig. 4. Realistic zinc model of the extension to the Berlin Museum with the Jewish Museum Department. ©SDL. Image courtesy of Studio Daniel Libeskind

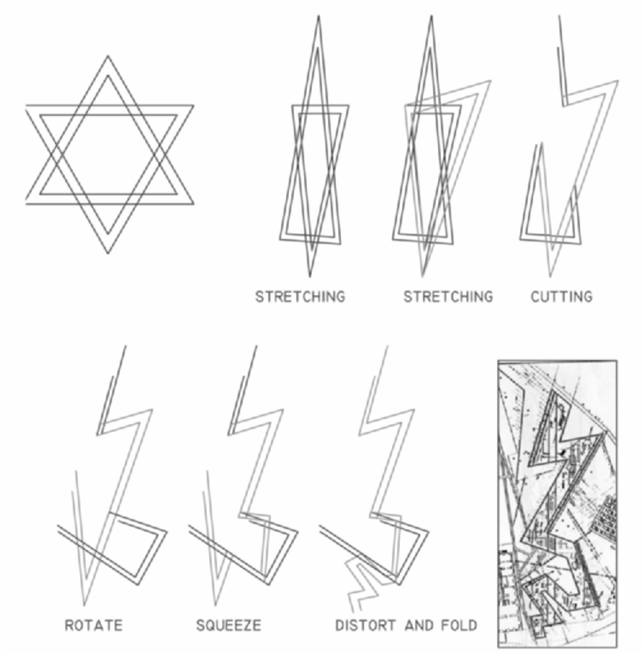

Fig. 5. Topological transformation of the Star of David in the plan of Libeskind's Berlin Museum with the Jewish Museum Department. Drawing by the author.

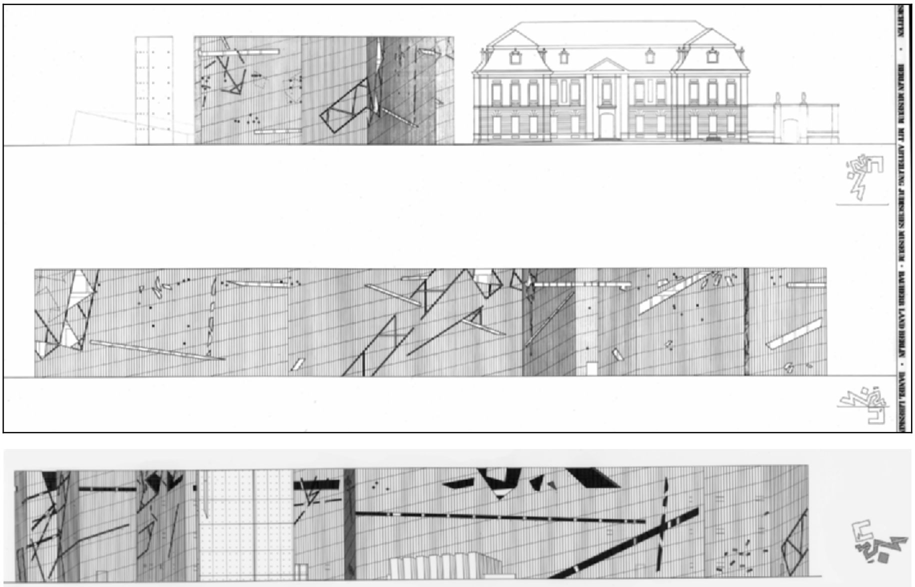

Fig. 6. Elevations of the extension to the Berlin Museum with the Jewish Museum Department. ©SDL. Image courtesy of Studio Daniel Libeskind

The dramatic zigzag, cut by oblique beams of light coming from slits in the perimeter walls, regulates the sequence of the expository sections in the only order possible for this space of contradictions, revealing the invisible and giving voice to silence (fig. 6).

The figurative effect is that of an architecture that, as Libeskind says, is "reduced to a sign of its absence" [Libeskind 1983]. As an extreme expression of the contemporary work it is pursued at various levels:

- the adoption of metallic surfaces for the outside hull that reflect the images of the surroundings, immaterialize the walled masses which, in contrast, are characterized by the consistent prevailing of solids over voids;
- the definition of the design of the elevations, whose punctuation dots, defined by the same graphic matrix of the cryptic "Chamber Works" are no longer windows to look out of, but non-oriented slits that permit of blades of light to enter like non-articulated screams in the hollows of the holocaust museum;
- in the declared reference to the dodecaphonic music presented in its final expression of an ineluctable reduction to silence, which is physically perceived in Schönberg's *Moses und Aaron*. The alternation of instrumental and vocal music as the maximum rarefaction of the body of sound immediately precedes the definitive dying out of the words, no longer sung, but spoken "o Wort, du Wort." With these eloquent monosyllables that create the figurative image of the death of every possible expression, the work, reduced to silence, can not but cease at the second act.[2]

The non-musical realization of the word as used by Schönberg, would be the respective voice of the non-architectural symbol as experimented in “Chamber Works” and reiterated in the Berlin extension, both burdened by numerous symbolic accents, both relative to each composed construction in itself and involving the destiny of the artistic expression in a more general sense, as well as in the interpretation of the theme of remembrance.

In this particular case, the architect quite diffusely recounted (see, for instance “Between the lines” in [Libeskind 1997]) all the reflections and personal studies on the musical work that he transferred to the project. Therefore, the physical space and sound are in a reciprocal relationship because the one inspired the other, even if it depends on constitutive laws that are not easily shown to correspond, with the exception of the particular association of the deconstructive style with the dissolution of the order introduced by the dodecaphonic music. This is not so much a generic study of chaos as it is an interpretation of disharmony as a new order, different from that of the classical order where harmonic laws dominate.

Music as Image

Peter Cook is the author of design experiences imbued in ideals that began with the historical group Archigram.

In the early 1980s he was involved in transferring the graphic form of Ernest Bloch’s concert for violin into the composition of the plan of an ideal city:

> A simple exercise was the interpretation of a piece of a violin concerto by Ernest Bloch. Not a piece that I know, but one that looked tempting on paper. The notes become towers, the stave becomes a street, the supporting markings become walls. Around the time, I had set a series of short projects for students on idea of “music” as a direct architecture … [Cook 1992].

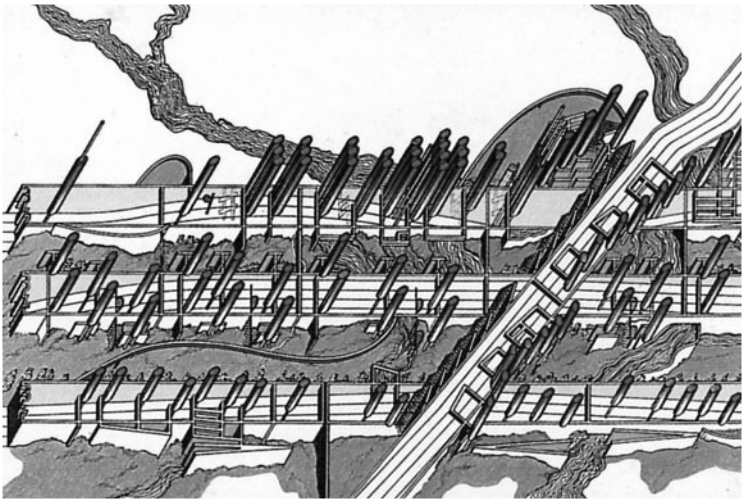

Fig. 7. Bloch City with towers arranged on musical staffs. Image courtesy of Peter Cook

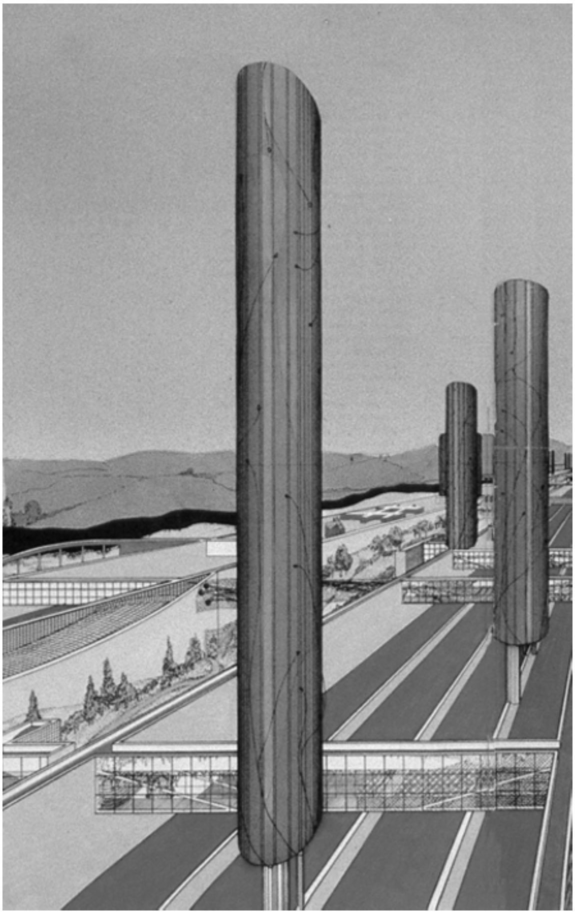

Fig. 8. Street view of Block City. Image courtesy of Peter Cook

Fig. 9. The measures of the concerto by Bloch that provided the layout for Peter Cook's Bloch City. Image courtesy of Peter Cook.

The notes on the staff represent the planimetric position of tall cylindrical skyscrapers while the musical lines are the urban highways of the "great march" [Cook 1985] (figs. 7, 8, 9).

As support for the melody, the extension of the staff is virtually infinite, and represents the basis of the fluent character of verbal and sound expression: it provides a road in space and time to the completion of the musical experience.

The spatial and temporal continuation, acquired in architecture as an evolution of perspective representation that fixes only an instant of the perception, is commonly obtained by using the composed figure of a path along which knowledge of space is accumulated as a succession of events. Rather than an indistinct, empty place, the road is the urban connection that confers continuity even in the presence of strong discretizing elements of caesura.

Bloch City's urban highways are three parallel staffs, cut diagonally by a fourth that appears as a link with some other place outside this system, which remains open and extensible both in the horizontal direction for its possible transcription of the whole concert, but also in the vertical, hypothesizing the representation of the entire polyphonic body of the orchestra.

From the geometric-mathematical point of view, we can see that internally the system is entirely lacking in continuous solutions, while it tends toward an external left and right limit with regard to the written beginning and end of the physical space, represented by silence.

The substantial disregard of this project for the natural and, in the final analysis, hypothetical context, strongly underlines the idea that the musical *continuous* and the spatial *continuous* of architecture both have the same nature, described by the same discrete graphical elements, by the same punctuation marks, by the same syntactic colouring. This is evident in this case because the project re-proposes an identical architectural and musical graphic composition, but it also makes a limit case evident in the correspondence that is obviously realized in the common compositional writing in both fields by using the traits that are characteristic of each.

The dividing "bars" of the beats are represented as bridges, rhythmic separations that acknowledge the units of space and time by establishing a correspondence between the musical measure – the beat – and the formal urban unit – the neighborhood.

The notes are re-interpreted as towers in a knowing formal transposition: it is in fact possible to note that this concert piece is full of chords and triplets that constitute the vertical structure or harmony of the text. In contrast to the continuous horizontal movement of the melody, even from a figurative point of view, Bloch City's tall buildings clearly represent the vertical aspect of harmony; they also reproduce the tall building's discrete and punctual form and the notes united in chords.

The symbol of expression repeated on all the accompanying chords (left hand of the script for the piano, lower staff in the figure) indicates a *tenuto* sound, meaning that the notes have to be held for the entire length of its value and also slightly accentuated: this fact highlights a formal discontinuity in the execution on the piano of a succession of harmonic sounds which is well expressed in the project of the isolated figures of cylindrical towers whose form, other than being the same as the notes in their modern representation – no longer quadrangular as is found with the ancient *neum* – is, in itself, closed, repelling, concluded; they do not formally admit any continuity with other geometric figures.

The connecting symbol, superimposed on the staff, is the indication that reunites this discontinuity, denoting the need to "play" the indicated piece as a whole. The continuous linear buildings, containing offices and studios that trace the form and the position of the buildings are therefore syntactically and formally precise.

Music as Method

This line of study, undertaken on an ideal level by Peter Cook, bringing suggestive ideas that derive from other horizons to the field of architecture, is also pursued and verified in the concrete reality of Steven Holl's construction in a home in the green country spaces of Texas that interprets the continuous and discontinuous tonal properties of the horizontal and vertical structures.

The Texas Stretto House condenses the temporal and spatial scansion of one of Bèla Bartòk's most singular concerts, "Music for String Instruments, Percussions and Celesta" [Bartòk 1937] (fig. 10).

The composition in four movements presents a clear distinction between heavy, discontinuous percussion elements and the lighter string elements, where sound flows without interruption.

Fig. 10. Bars from "Music for String Instruments, Percussions and Celesta" [Bartòk 1937]. Mirrored form. Drawing by the author

The building, made up of four distinct yet related parts, represents the structural co-presence of alternating heavy and light elements: heavy orthogonal walls, primarily containing the service areas, and light curved metallic roofs connecting them.

The inversion of the theme in the work by Bartòk, not so much for the *fugata* structure as for the tonal substance, is the beginning of the second movement; the violins assume a discrete character with their *pizzicato*[3] entrance, while the piano, a chord instrument that is struck, comes in with a melodic phrase, connected; an inversion which is well expressed in the guest house, an autonomous volume where the walls become soft, curved, continuous, while the roof becomes heavy and material.

However, these perceptive similarities are only the most evident traits of a precise, constant and knowledgeable transcription of the musical work into the architectural design, a composition where the opposing themes of continuity and discontinuity are already expressed in their entirety in the first movement, and are repeated obsessively (and on different levels) throughout the course of all four movements.

It is interesting above all to observe how the *stretto* modality is pursued by sliding the entrances of the *fugato* theme up until the re-uniting finale.

For the entire length of the piece, the *fugato* style is manifested with an increasing intensity up until the entrance of the celesta, an architectural instrument which Bartòk used to explore compositions regulated by the golden ratio, to arrive at a successive, sudden decrease in the intensity in the finale, where the theme, only slightly modified, is presented in a mirrored form, going back to the original tonality.

The Stretto House travels the same temporal and spatial course: the sliding of the entrances can be read in the passages created in the walled masses; the architectural thickening of the central part corresponds to the orchestral thickening/broadening obtained by the gradual entrance of an ever increasing number of instruments; the central insertion in one of the most intense movements of the physical image of the harmonic relationships that define the cut of the windows in the two lateral views, recalls the entrance of the celesta where the tonal passages are of interest – the "distances" between one note and another – corresponding to the measure of the geometric relationships that run between the rectangles set in the windows (fig. 11).

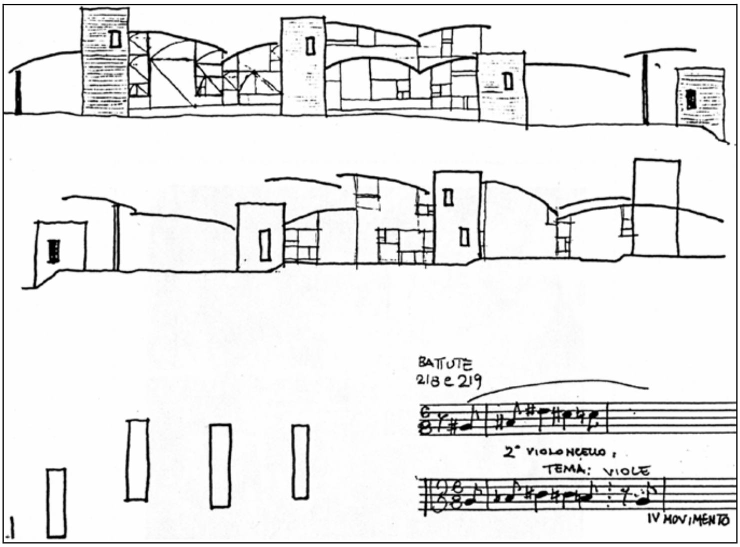

Fig. 11. Comparison of Bartòk's specular composition and Holl's Stretto House.
Drawing by the author

The rapid emptying of the built masses that ends with the symbolic hollowness of the last patio on the top floor, where a pool and a flooded room completely reflect the thematic structure, giving an identical ending to the composition. The final walled element visualizes the others, imprisoned in the construction, and in its reflection, it depicts the theme in its entirety.

Formally, the internal construction of the two works are of great interest, because they are both proportioned according to the golden ratio (fig. 12).

The study of these harmonic measures runs throughout the entire oeuvre of Bartòk and is identifiable in "Microcosmos," an academic compilation for piano, as well as a particular exercise composed at various levels in "Music for String Instruments, Percussions and Celesta."

The 89 measures in the first movement are divided in two golden segments: from the gradual crescendo, from the *pianissimo* in the beginning of the composition reaches the *fortissimo* apex at the end of bar 55, to the *diminuendo* which leads to *pianissimo* after the last 34 bars, to the more internal subdivision in golden measures of the timbered pushing and releasing the *soft pedal*, executed in decreasing progression; bars 34-21, 21-13, 13-8.[4]

In Steven Holl's house, a first division in anthropometrical measures, (expressed in the foot/inch system), of the areas within the blocks of walls, is simply repeated in the voids and solids on both the ground and upper floors, and recalls the measures 21+13 feet that are typical of the Fibonacci series. A timbered internal division of the solids and voids of the main façade is easily deduced by the golden drafting process of the decomposition as studied in Hanning, known to us by the rich epistolary rapport with Le Corbusier. It is also possible to see a more general harmonic disposition of the glass walls composed of rectangular windows.

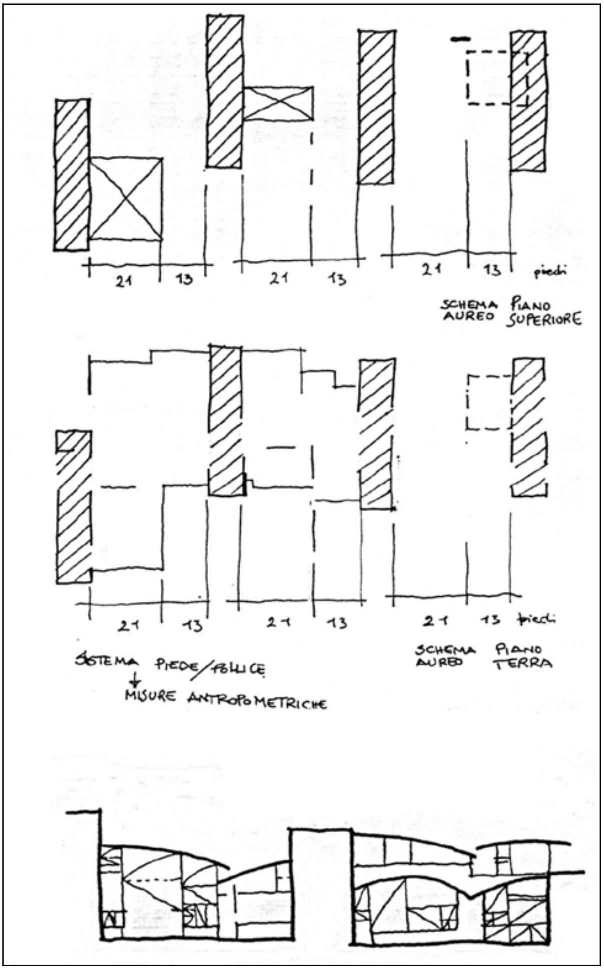

Fig. 12. Identification of the golden section in the plan and elevations of the Stretto House. Drawing by the author

A first effect is created by making the slight variations in the quota of the vertical section coincide with the inclines and declines of the *fugata*: this coincidence becomes rigorous by simply "counting" in the first passage, the different levels (that go up and down) from the entrance to the living room and the art gallery, and another in the second, in the interior where there is another inversion to reach the study/studio.

Even the voices, in the *fugata*, show an alternation of inclines and declines, in the entrances: the second, the fourth, the sixth and the eighth, … each enters a fifth higher than the last, while the third, the fifth and the seventh, all go down a fifth.

Finally, the arrangement of the rectangular walls is not left to chance: the reciprocal position corresponds graphically exactly to the beat as it is found in the figure that is expressly cited by Steven Holl in a watercolour connected to this project.

The beat in question is an ulterior re-proposal of the main theme, performed in this case by the second cello.

There is one last observation to make regarding the superimposition of the longitudinal views of the curving roofs which lie on different floors: the figure reminds us of the compostion of different musical phrases produced contemporarily by the various instruments. In the continuous counterpointing of the "Music for String Instruments, Percussions and Celesta," the voices of different violins and other strings overlap, each occupied with the execution of its own piece. In the continuous counterpointing of the Texas house, the timbre of the architecture is realized synchronically as though to sew it back together using the symbolic roofs in the form of a musical bond made tri-dimensional; the relentless discontinuous presence of the walled boxes.

Multimedia Architecture[5]

We can see that the most evident characteristic linking the projects that have been discussed here is their close relationship to music. The nature of their reciprocal bonds, as well as the reasons behind their mutual connections, are quite diverse. They substantially belong to two broad categories. The first of these tends to establish similarities or even equivalents in the exterior expression by imitating the formal characteristics. It is therefore a form of translation from one language to another.

Instead, the second category contains works that acknowledge the same compositional rules. These are not always easily identifiable if the observation is limited to their exterior aspects, and because of this it is necessary to look deeper, at the compositional structure, ordering concepts, and structural logic, and to see if, and to what degree, these elements are the same in both the architectural and the musical works.

In the different cases, the cause of these correspondence is sometimes believed to depend on the mutual dependence of the logical/compositional structure deriving from mathematical laws, intended not so much as a body of rules aimed at calculation but as general criteria to give logical sense to the procedures of organized thought.

Each composition, whether it be musical, architectural, or artistic in the broad sense – comprising the most extreme fringes of abstractionism – consists in identifying one theme, in its simple or complex repetition, by using the technique of variation, in the intentional ordered or casual arrangement, continuous or discontinuous, of the different elements in the final constitution of this complex operation into an entity that is the complete work.

The physical effects, the similarities, are more immediately recognized than the demonstration of their logical, mathematical origins.

A very useful tool for this, but also for a more precise verification on what has been expressed here with concepts that are not always easily discernible, is the computer utilized to elaborate hypertext structures able to take advantage of its simultaneous potential and multimediality in the analyses of those architectural structures that may be defined as multimedia.

The musical pieces to which the designers refer are not always easily identifiable: we find ourselves in the position of having to accept the designer's declarations, which are sometimes full of poetic references.

The process of comparative analysis is therefore long and sometimes lacking in objectivity. The simultaneousness of seeing and hearing that which is created by a hypertext instrument is therefore necessary as a means of comparison.

This method has been adopted for the Texas Stretto House. The brief hypertext which was realized includes all of the elements of study identified, compared and described above with regard to Steven Holl's work (figs. 13, 14, 15).

The result seems more clearly exhibited, and most of all, accessible even to those who are not accustomed to staffs, eighth-, sixteenth-, thirty-second- and sixty-fourth-notes, to musical timbres and harmonic constructions, disharmonies, dissonance, thematic inversions, and all that is part of the musical lexicon. This form of expression is sometimes verbally very similar to that which is used in an architect's common language, rich in metaphors, metonymies, meta-languages and every sort of symbolism and speculation that tend to the limits of abstraction.

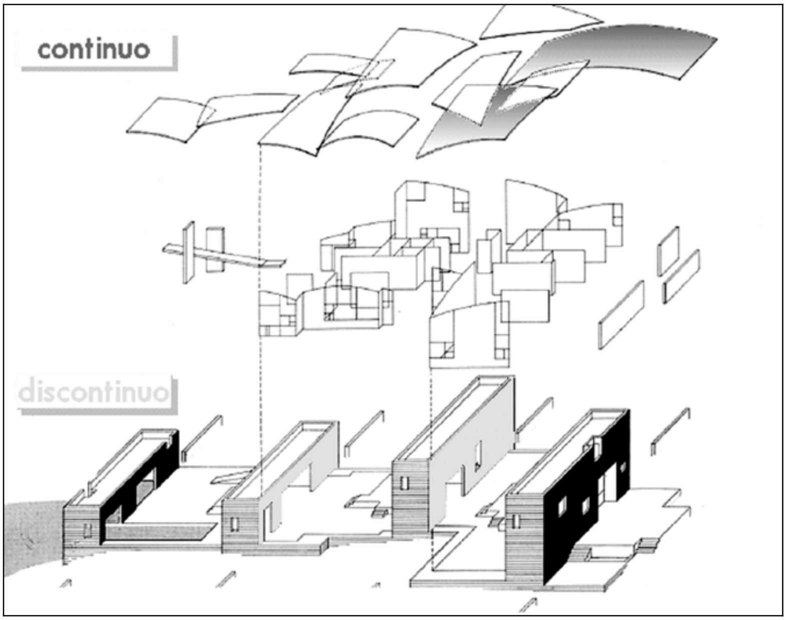

Fig. 13. Extracts from the multimedia analysis of Holl's Stretto House. Slide by the author

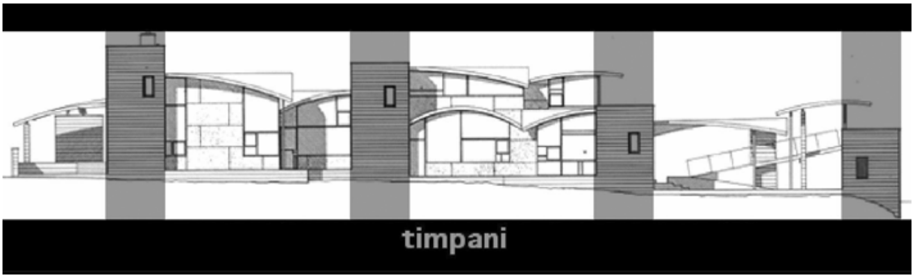

Fig. 14. Extracts from the multimedia analysis of Holl's Stretto House. Heavy elements – orthogonal brick walls – kettledrums. Slide by the author

Fig. 15. Extracts from the multimedia analysis of Holl's Stretto House. Light elements – metallic roofs – violins. Slide by the author

Notes

1. A set of twenty-eight drawings done by Daniel Libeskind while he was the head of the Architecture Department at the Cranbrook Academy of Art in Bloomfield Hills, Michigan.
2. The double identification of the reduction to silence by using the term "word", which in the German language is monosyllabic, is not banal. On one hand, the evident linguistic symbolism pursued in the reduction of the number and the variety of phonemes in a quantitative "diminuendo" expressed the impossibility to communicate, on the other, the analogy that is not exquisitely formal, but structural, with the way in which most of the musical compositions are concluded – defined by Libeskind as a reduction to silence – that often occurs by reiterating the note that returns to the tonal theme which alone, plays for the entire length of the last beats, tied into a singular, last and final afflatus.
3. The "Bartòk pizzicato" consists of elevating the chord and then suddenly releasing it; the effect thus obtained is more intense than the light pizzicato that is commonly used by playing the chords of the strings as you would a guitar.
4. Explicit referral to the Fibonacci succession 1:1:2:3:5:8:13:21:34:55:89… is found in [Szabolcis 1956].
5. The multimedia presentation was a *Macromedia Director* production, originally created in 1995 with an Apple system Mac, now "unreadable", but "translated" into a more static *PowerPoint* presented at the recent conference "Sound and Image", Aula Magna in the Facoltà di Medicina, 30 May 2008.

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Alessandra Capanna is an Italian architect living and working in Rome. She has taken her degree in Architecture at University of Rome "La Sapienza", from which she also received her Ph.D, with a thesis entitled "Strutture Matematiche della Composizione", concerning the logical paradigms in music and in architecture. She is the author of *Le Corbusier. Padiglione Philips, Bruxelles* (Turin, 2000), on the correspondence between the geometry of hyperbolic paraboloids and technical and acoustic needs, and its final and aesthetics consequences. Among her published articles on mathematical principles both in music and in architecture are "Una struttura matematica della composizione", remarking the idea of self-similarity in composition; "Musica e Architettura. Tra ispirazione e metodo", about three architectures by Steven Holl, Peter Cook and Daniel Libeskind; and "Iannis Xenakis. Combinazioni compositive senza limiti", taken from a lecture given at the Dipartimento di Progettazione Architettonica e Urbana at the University of Rome. She is teacher at the First Faculty of Architecture of Rome "La Sapienza" . She has taken part to Nexus Conferences III and VI speaking about "Conoids and Hyperbolic Paraboloids in Le Corbusier's Philips Pavilion" (2000) and "BiOrganic Design. A New Method for Architecture and the City" (2006). She is a member of the editorial board of the *Nexus Network Journal.*

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Research

Using Key Diagrams to Design and Construct Roman Geometric Mosaics?

Abstract. The complexity shown by some geometrical patterns of Roman mosaics and the high quality of their realization lead to think that for such patterns, unlike scenes with human or animal figures, a model of the general pattern was certainly not sufficient to guide the setting up; in order to answer this question one is led to conjecture the existence of diagrams (key diagrams) with which the craftsman, by looking at them, is able to identify (and/or remember) the geometrical structure of a basic element of the general pattern, as well as a way for constructing it – and possibly the whole pavement – with his usual instruments. This hypothesis is applied to some patterns which were well spread over the Roman world. The present study aims at showing how a given key diagram can apply to varied patterns and, conversely, how the making of a given complex pattern can rely on several articulated key diagrams.

Introduction

Archaeologists who specialize in the study of mosaics generally do not show much interest in "geometric" ones, and prefer to focus their research on those representing "living" scenes, i.e., those featuring gods, humans or animals. However, geometric mosaics are not devoid of interest, since they may give insights into the craftsmen's geometrical knowledge and processes,[1] from which it is sometimes possible to ascertain links between practical and theoretical knowledge. Based on my personal experience [Parzysz 2007], I would advocate that mathematicians interested in the history and the epistemology of geometry work with archaeologists, in order to increase our knowledge about these points and to help them see facts that they might otherwise miss. And, in a more practical way, this can also lead to making better restorations of damaged mosaics (see for instance [Bar-Shay 1995: 124]). Of course, the geometry involved in such studies is quite elementary, of the type taught at high school level, but studying geometrical mosaics from a geometrical point of view – which seems an obvious thing to do – has not aroused much interest until now, even if some attempts must be mentioned, such as [Prud'homme 1975], [Tebby 1994], [Hanoune 1994] and [Bar-Shay 1995, 2005].

In this paper I would like to deal with a general question about mosaics: starting from the fact that identical geometrical patterns – which are sometimes quite complex – were made through time and space in the Roman world, one comes to the conclusion that some means for passing them on from one craftsman (*pictor*) to another necessarily existed. But, since there is only little archaeological evidence of this phenomenon,[2] we are bound to make hypotheses about what these means were, and in this paper I will develop one such hypothesis on some chosen examples (among many others).

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1 Posing the problem

With regard to mosaics with figures, some specialists think that Roman mosaicists probably made use of sketchbooks of models that served a double purpose: on the one hand, they enabled prospective customers to choose their favourite subjects; on the other hand, they could be used as models for reproducing patterns. Thus, for instance:

> It is almost certain that, at the time we are concerned with, mosaics with figures were neither original works nor real creations. They were products which were copied, modified, adapted, or composed from sketchbooks in possession of the mosaic workshops, from which the customer selected the subjects he wanted to be represented in his home [Daszewski & Michaelides 1989: 14] (my trans.).

Others disagree with that point of view and think that there were no such sketchbooks. For instance,

> Nobody has ever seen any sketchbook and no text or image can vouch for their existence [Bruneau 1987: 154] (my trans.).
>
> The "sketchbooks" hypothesis is of no use: it is not necessary to suppose their existence to explain the passing on of the iconographic tradition [Balmelle & Darmon 1986: 246] (my trans.).

Nonetheless, the fact remains that the passing on of an iconographic tradition has taken place through time and space, either in the form of sketchbooks or of teaching in the "guild" type:

> It seems to us that the main part of the passing on must have taken place in a living way by a passing on from master to disciple, during the pictor's training [Balmelle & Darmon 1986: 247] (my trans.).

At any rate, the existence of such a passing on appears as the most important element, whatever the vectors used to maintain it.

Regarding the geometric mosaics, let us now consider the following question: How is it possible, on the sole basis of a picture, either drawn or mental, to be able to realize a geometric mosaic with a complex pattern in a satisfactory way?

If one is provided with a drawn representation, the first question is about its nature: it may be a precise drawing, made with instruments, or a mere sketch made only by hand, like those on the medieval architect Villard de Honnecourt's sketchbook [Erlande-Brandebourg et al. 1986]. If it is a mental image, it will be necessary to concretise it on the ground with the usual instruments: line and ruler, and perhaps – but not necessarily – square and compass.

In the case of a precise scale drawing the first idea which comes to mind is to reproduce it at real size on this sole basis. Then a double problem arises, linked both with scale and reproduction. At first sight, the question of shifting to real size, from measures on the drawing or by reading dimensions, can be solved in two different ways: either with a calculation of proportionality or by making use of a proportion scale. It is well known that the Roman numeration system, since it is not positional, is not very good for calculation and implied the use of *abaci*, especially when multiplications and divisions were needed, as in this case. That is why making use of a proportion scale is more likely. But this technique makes three operations necessary for each length:

1. measuring the original length in the drawing;
2. transforming it with the scale;
3. transferring this new length *in situ*.

Besides the tediousness of this process, each one of the three above operations generates some kind of imprecision:

1. (*measurement*) imprecision of the drawing and of the instrument with which the length will be transferred;
2. (*transformation*) imprecision of the scale, of the transfer on the scale, of the reading;
3. (*transfer*) imprecision of the instrument.

Used this way, this technique would give an inaccurate result, but to some extent this drawback could be contained with a guiding grid:

> In marking land divisions, cumulative error was prevented by establishing a taut grid framework, which was done either by subdividing the surface from equidistant points along vertical and horizontal axes, that is working from the centre outwards, or by dividing the perimeter of a square or rectangle into equal units and connecting the points on opposite sides, that is working from the outside inwards. The latter procedure would be more practicable for laying out a mosaic pavement [Tebby 1987: 277].

Nevertheless, such a "servile" method will quickly show its limits, because of some internal geometrical relations between the components of the pattern. Here is an example taken from a "multiple pattern" mosaic found in Migennes, France [Bassier, Darmon & Tainturier 1981]. These two square panels (figs. 1, 2) are basically filled with the same pattern, described as a "star of eight lozenges inscribed in a circle" [Balmelle et al. 2002; pl. 289].

Fig. 1

Fig. 2

This description is formal, and it does not quite fit the real mosaic, since the shapes of the stars on the two panels are not identical, and the so-called "lozenges" look more like rhomboids. The corresponding theoretical pattern (fig. 3) results from a simple geometrical construction beginning with that of a regular octagon inscribed in a circle, then linking every third vertex in order to obtain a star octagon (fig. 4).

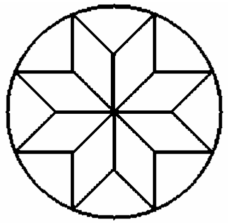

Fig. 3.[3]

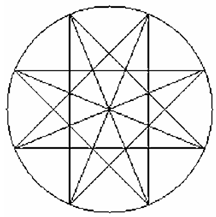

Fig. 4.

It seems obvious that this is not the process implemented in Migennes, and we can say that, when designing the pattern, the craftsman was not conscious of the subjacent star octagon: he contented himself with putting eight adjacent quadrilaterals around the centre of the circle. This is far from being always the case: see for instance fig. 5, from Adana, Turkey [Balmelle et al. 2002: pl. 289c], on which the alignments are much better respected. Anyhow, and beyond any supposed "clumsiness" of the mosaicist, this example shows how the difference between the craftsman's having a geometrical or a non-geometrical view of the pattern can be detected.

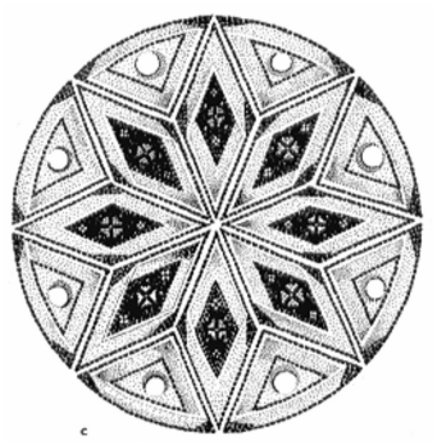

Fig. 5

Another process can be thought of, which is both easier to use and more efficient than just using a guiding net. This process is essentially similar to the one used "naturally" in the case of scenes with human or animal figures, that is:

– identifying *signifying elements* (figures, parts of the body…)

– discerning an *internal logic* in the work, in order to articulate these elements (composition of the scene, disposition of the body parts…).

For a geometrical pattern, the basic unit will be an element for which a geometrical structure and a possible construction process will have to be determined. And, in case this element is duplicated, a repetitive pattern (i.e., a transformation group) will have to be identified.

However, if a gifted craftsman can easily reproduce by mere sight a subject with human or animal figures, the complexity of some geometrical patterns – which had to fit within a given surface (rectangle, square, circle…) – made it difficult to set them up by just imitating the finished pattern. Of course, as we have seen above, mosaics for which you can say the craftsman did not really understand the geometrical structure can be found, because, in comparison with a theoretical model, they show irregularities that cannot be attributed only to the workman who laid the *tesserae*. In a geometrical pattern, any property not taken into account will result in a small deviation which, once repeated throughout the mosaic, will increase and possibly make a makeshift repair necessary in some place, as can sometimes be observed. From this, and the fact that some geometrical mosaics have a perfect design, one is led to conjecture that the *pictor* had a good geometrical knowledge, including in particular one or several diagrams, either mental or drawn, of the basic element of the pattern, allowing him to undertake a geometrical construction of this element as well as of the global pattern. I will call such a diagram a *key diagram*. When drawn, a key diagram does not necessarily have to be made with great precision; it may even be drawn by hand, the condition being that it has to be "readable", i.e., that by looking at it you are able to see its geometrical properties and a way to construct it with simple instruments. The existence of key diagrams, which are situated at a junction point between learned and practical geometries, is of course a hypothesis, but it explains, on the one hand how complex patterns could be obtained from simple diagrams, and on the other hand how some mosaics could attain an astounding degree of complexity and precision. Some quite common key diagrams could be learnt and memorized by the *pictor* during his training, in the same way as high school students learn and memorize how to draw a perpendicular bisector with ruler and compass nowadays. Some other, more specific complex patterns, could result from a personal elaboration and possibly be noted down if the mosaicist, finding them nice and interesting, wished to be able to re-use them later. Moreover, as we shall see, a key diagram could be used, through variations, to produce different patterns.

In order to illustrate this, I will give here below some examples based on two elementary figures inscribed in a square (a tilted square and a regular octagon), together with their use in the making of some complex geometrical patterns.

2 Tilted square within a square

This part is about a pattern which can be found in France, both in the southwest (Montcaret) and in the northeast (Metz, Liéhon), and also in Germany (Kreuzweingarten). These mosaics are not quite identical – they differ from one another in their ornamental details – and not contemporaneous (they were dated between the second and the fourth centuries A.D.), but their geometrical structure is quite the same (fig. 6).

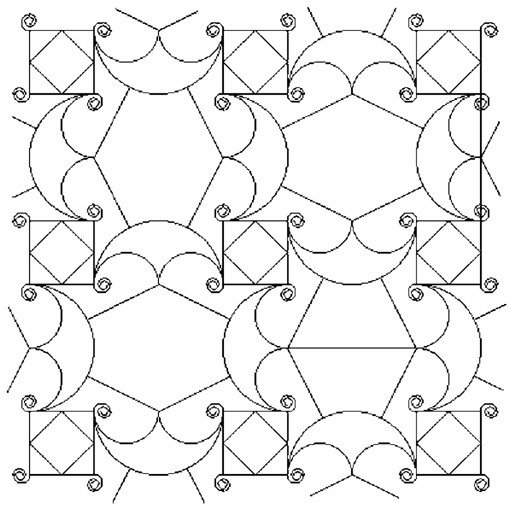

Fig. 6.

This pattern has been described as “a geometrical pattern […] of *peltae*[4] (Amazons’ shield) […] inserted in an outline of imbricated lozenges”[5] ([INRAP 2005], describing the mosaic of Liéhon).

An first examination of this pattern shows that it is built on a square grid (fig. 7). A more thorough investigation reveals a new structure, made of lozenges and squares (fig. 8).

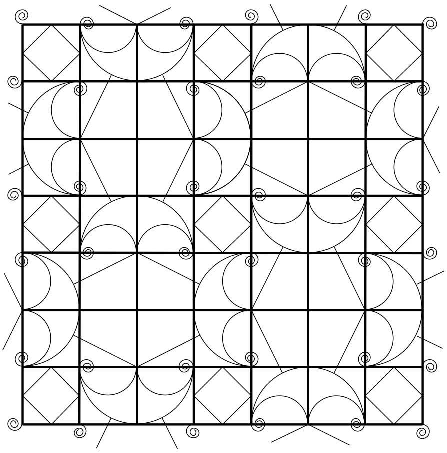

Fig. 7.

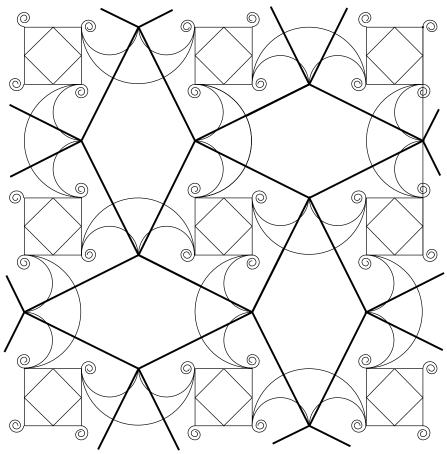

Fig. 8.

Setting the *peltae* and their volutes aside, a basic element of the pattern, which is easily obtained by dividing the sides of a square into three parts, can be identified (fig. 9).

The composition of this pattern, as well as a way for constructing it, can easily be read on the diagram. The lozenges appear by reflections, with sides of the square as axes of symmetry; iterating such symmetries will allow the setting up of the pavement (fig. 10).

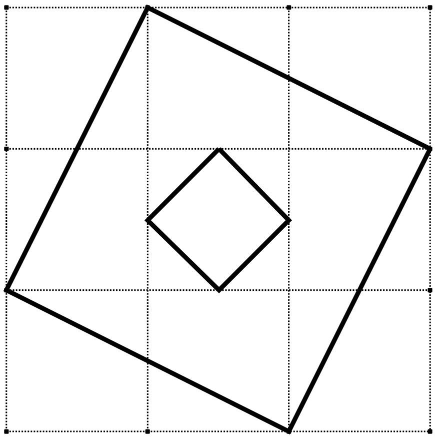

Fig. 9.

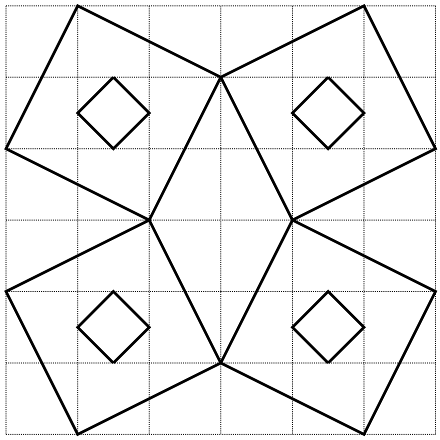

Fig. 10.

On this basis, the setting up of the pattern can be operated in three steps:

1. Drawing the square grid (fig. 11);
2. Drawing the tilted squares within "blocks" of 3×3 unit squares, and small "poised squares" in the centre of each block (fig. 12) by using symmetries with lines of the grid as axes;
3. Drawing the *peltae* with half-circles constructed from knots of the grid (fig. 13).

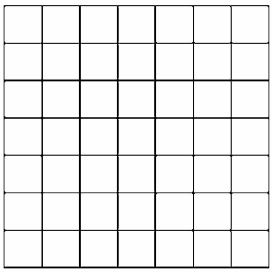

Fig. 11.

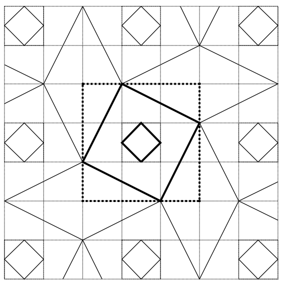

Fig. 12.

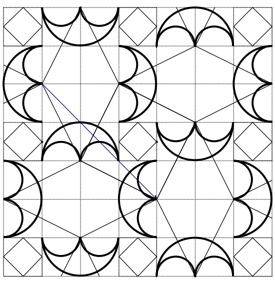

Fig. 13.

Then all that is left to do is to draw (freehand) the volutes at the extremities of the *peltae*.

The drawing in fig. 10 makes it possible to carry out the first two steps of the construction, because it indicates not only the position of the tilted squares in the square grid but also their relative positions (symmetries). This is the reason why I consider it as a key diagram for the pattern in fig. 6. We may also remark that by itself this diagram makes up the pattern of a mosaic pavement, as for instance in Lyon, France, and Itálica, Spain [Balmelle et al. 2002: pl. 408a, b].

As for the *peltae* and their volutes, they can be considered as decorative elements, added in order to adorn with curves the otherwise stern rectilinear pattern. As mentioned above, *peltae* are a common design in Roman geometric mosaics and can be associated with various polygons (in particular squares, rectangles and lozenges).

3 Regular octagon inscribed in a square

3.1. Constructions

The regular octagon inscribed in a square (fig. 14) is a design frequently used in Roman geometric mosaics. Two "classical" construction processes – at least – can be thought of. They are represented in fig. 15 and fig. 16, which can be "read" easily and detailed below, and used as key diagrams.

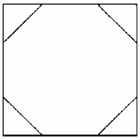

Fig. 14

First process

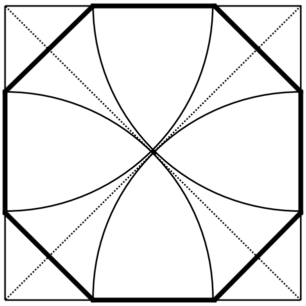

fig. 15

1. In a square, draw the two diagonals.
2. Draw the four arcs of circle centred at the vertices of the square and going through its centre.
3. These four arcs intersect the sides of the square at eight points, which are the vertices of the octagon.

Second process

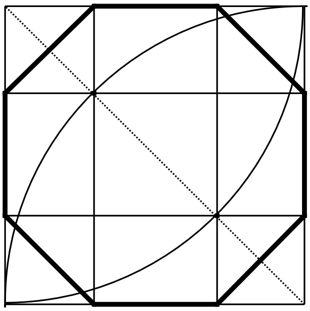

fig. 16

1. In a square, draw a diagonal.
2. Draw two arcs of circle centred at the extremities of this diagonal and going through the other two vertices of the square.
3. The parallels to the vertices of the square going through the points of intersection of the arcs with the diagonal intersect the sides of the square in eight points, which are the vertices of the octagon.

One is led to think that such construction processes could be memorised under the form of a diagram associated with their making, this diagram being *de facto* a key diagram of the corresponding configuration.

3.2. Eight-lozenge star

Let us now consider the pattern represented in fig. 17, which is widely spread throughout the Roman world. It is described as an "outlined orthogonal pattern of tangent eight-lozenge stars forming squares and smaller poised squares" [Balmelle et al. 2002: pl. 173b]. At first this pattern may appear complex, and one can easily understand that its accurate setting up might cause some problems to the craftsman. I will hereafter consider two possible "readings", both of them based on the octagon inscribed in a square. One may remark that this elementary pattern does not appear explicitly in the mosaic.

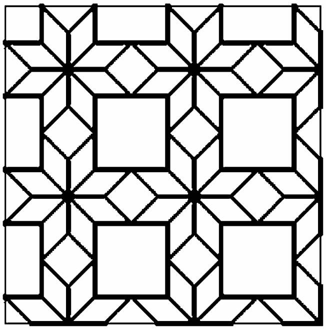

Fig. 17

4 Readings

4.1. First reading

Focussing on the so-called "stars", adjacent regular octagons can be distinguished. They delimit poised squares between them (figs. 18, 19) and constitute an "outlined orthogonal pattern of adjacent octagons (forming squares)" [Balmelle et al. 2002: pl. 163a].

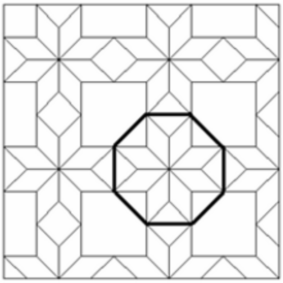

Fig. 18

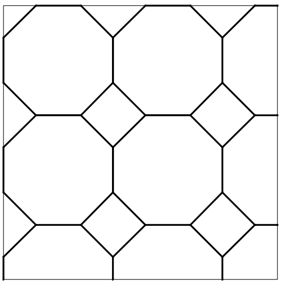

Fig. 19

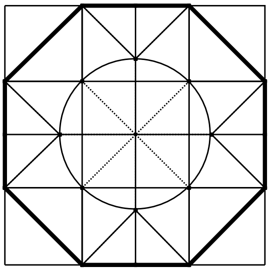

Fig. 20

Each star can be inscribed in a square belonging to a square grid, according to the key diagram of fig. 20, the making of which, starting from an octagon inscribed in a square, can be described as follows:

1. Draw the medians of the square;
2. Draw the circle going through the points where the diagonals of the octagon parallel to the sides of the square intersect.

Thus, all the points needed for drawing the star are constructed.

One just has to divide the surface of the pavement into squares and draw the key diagram of fig. 20 in one of the squares to be able to set up, by transferring some lengths, the orthogonal grid ("tartan grid")[6] with which the whole pattern will be drawn. This grid can be set up in two steps:

1. Setting up the tartan grid which will make it possible to draw the octagons (fig. 21);
2. Setting up the additional lines which will make it possible to draw the poised squares (fig. 22).

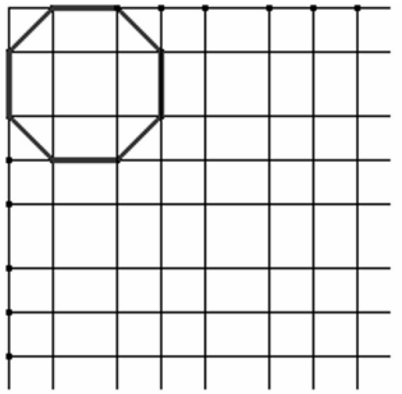

Fig. 21

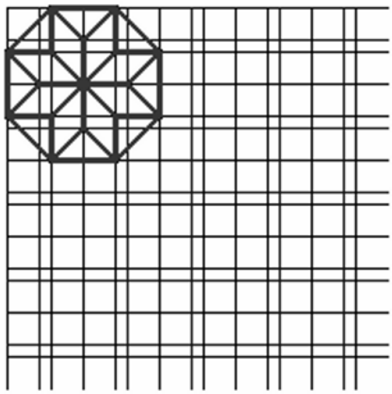

Fig. 22

Note. To set up the tartan grid, one may for instance realise key diagrams in the squares at the corners of the square grid, then draw lines joining corresponding points by lines parallel to the grid.

4.2. Second reading

Focusing now on the large squares in fig. 17, one may distinguish another orthogonal pattern, based also on regular octagons, but which are secant (figs. 23, 24).

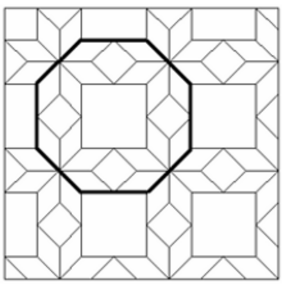

fig. 23

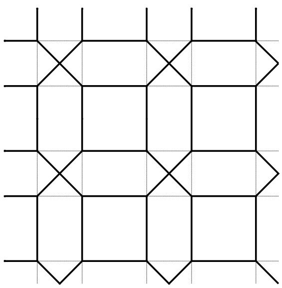

fig. 24

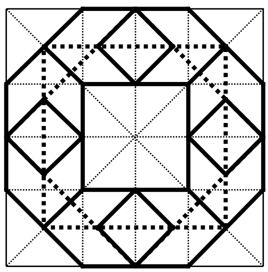

fig. 25

The new key diagram (fig. 25) starts from the octagon inscribed in a square as well. It can also be easily read:

1. Draw the diagonals and the medians of the square;
2. Draw two squares, the vertices of which are the middles of the sides of the octagon (thick dotted lines). This gives all the points necessary for drawing the pattern.

Note. This key diagram is more visible in mosaics which show a single octagon, shown in fig. 26, found in Bignor (Cornwall, UK) [Balmelle et al. 2002]: pl. 391b). The associated key diagram, quite apparent in the pavement, is clearly similar to fig. 25.

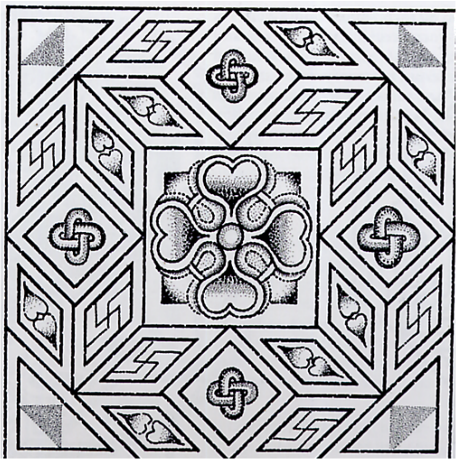

Fig. 26.

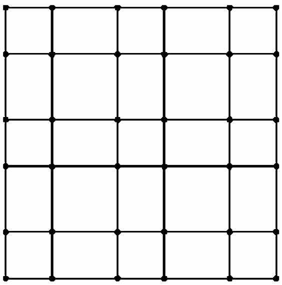

Fig. 27.

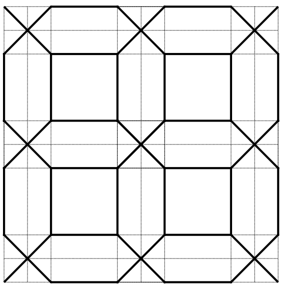

Fig. 28.

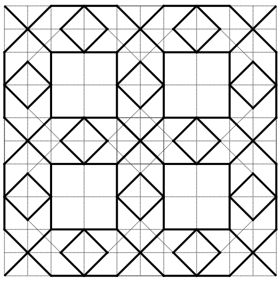

Fig. 29.

To set up the whole pattern on this basis it is possible, as with the first reading, to begin with the construction of the tartan grid associated with the octagon-in-a-square (fig. 27) and, after drawing the octagons, draw additional lines joining the intersections of neighbouring octagons (fig. 28). The pattern will then be completed with the medians of the squares circumscribed about the octagons, which will make it possible to draw the poised squares (fig. 29).

Note. A simplified – and frequent – variant of this pattern consists in replacing the first tartan grid by a regular square grid. Hence irregular octagons, and "lozenges" which are in fact parallelograms. In this case a single key diagram is sufficient to set up the whole pattern (fig. 30). But this form is not often associated with a pattern of eight-lozenge stars; it is more frequently used to get a pattern of secant octagons. In this case, the key diagram coincides with the pattern itself.

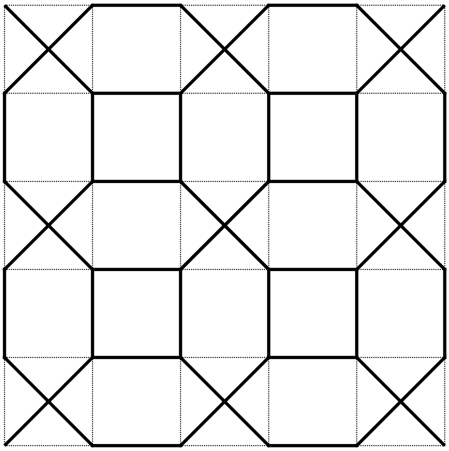

fig. 30

This example shows that several key diagrams can be associated with the same geometrical pattern. Since they are attested in simpler forms of the pattern (here: a pattern of adjacent or secant octagons), all these diagrams have some likeness. Moreover, in both cases the linking of two key diagrams (an octagon in a square followed by a set of octagons constructed on the tartan grid) allows the setting up, by a rather simple process, of a pattern which may look complex. Such a pattern could certainly not be obtained by just copying a drawing (even accurate) of the whole pavement, because it would not be possible – except by chance – to get regular octagons or lozenges.

5 Wreath-like pattern of squares in an octagon

A single key diagram can be found under different forms.

The pattern that we will now study (fig. 31) is described as "wreath-like pattern in an octagon and around a circle, of eight poised lateral squares, these motifs contiguous to each other, forming lozenges and creating the effect of an eight-pointed star" [Balmelle et al. 2002: pl. 315]. The associated key diagram stems directly from the key diagram in fig. 25.

The octagon-in-a-square being constructed, this diagram (fig. 32) can be read as this:

1. Draw the diagonals and the medians of the square;
2. Draw two squares, the vertices of which are the middles of the sides of the octagon (thick dotted lines);
3. From the intersecting points of these two squares, draw lines parallel to the sides and the diagonals of the initial square.

Thus, all the points needed for constructing the pattern are obtained.

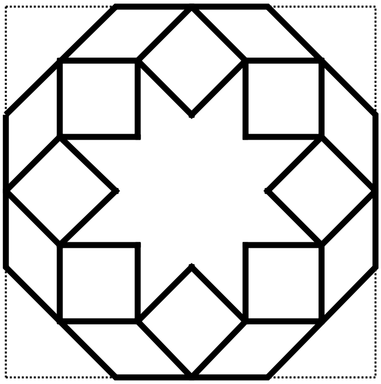

Fig. 31.

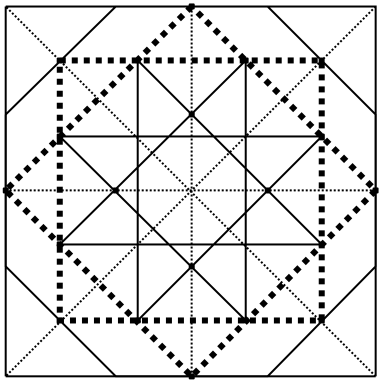

Fig. 32.

We can see that fig. 32 and fig. 25 stem from a common pattern, which is none other than fig. 33, composed of a regular octagon inscribed in a square and two squares, the vertices of which are the middles of the sides of the octagon.

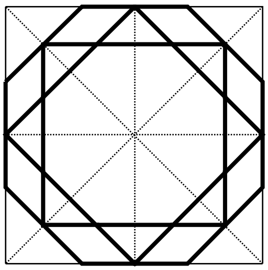

Fig. 33.

We may then consider fig. 33 as a "basic" diagram, which could possibly be completed in different ways to give rise to more "sophisticated" diagrams making it possible to achieve more specific patterns. This leads to conjecture the existence of several articulated levels of key diagrams of increasing complexity, such as, for instance:

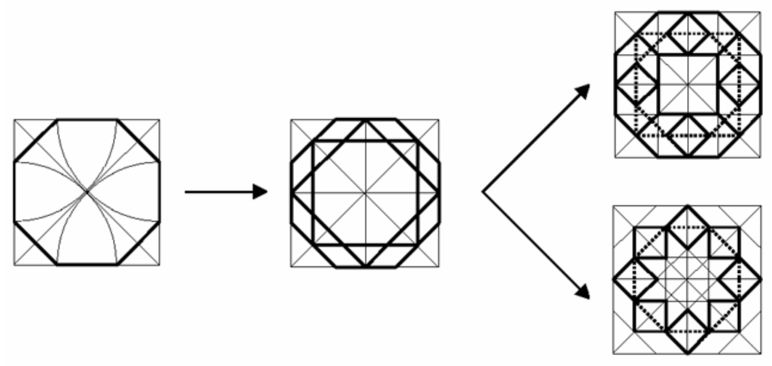

Fig. 34.

6 Conclusion

Studying the geometrical properties of a complex pattern leads, on the one hand, to identifying a basic element which is eventually replicated throughout the mosaic pavement by way of geometrical transformations (translations, symmetries, rotations…) and, on the other hand, to looking for possible constructions of this basic element with instruments which were commonplace when the mosaic was made. Such constructions had to be passed on from one craftsman to another under the form of a diagram on which the basic pattern, as well as its construction, is clearly "readable". This pattern may be pretty general (like the octagon-in-a-circle) or more specific (like the eight-lozenge star), and we have seen that a complex pattern can result from articulating several key diagrams of increasing complexity.

We have also seen that various key diagrams may possibly generate a single pattern, and conversely a single key diagram may be used in the construction of several patterns. The study of Roman mosaics shows that two mosaics which at first look identical are very seldom quite similar; most of the time, even if their constitutive elements are the same, their composition shows greater or lesser differences, as if they were local variations imagined by the *pictor* from a general basic formula. Nonetheless, without any archaeological evidence, which key diagrams were used to make a given mosaic can only be conjectural, as is the very reality of such diagrams, even if it seems quite likely.

Notes

1. For instance, in Besançon (France) a detailed geometrical study allowed me to explain that some apparent mistakes of the craftsmen could in fact be justified and how some real mistakes occurred [Parzysz 2007].
2. Some scarce traces of lines are sometimes found under a mosaic (see for instance [Chantriaux et al. 2007]), but they are far from being sufficient to give an accurate idea of how it was designed.
3. The diagrams of this article were made with the Cabri® II *Plus* software.
4. This is a very common pattern, consisting of three half-circles organized as in fig. 6.
5. We may remark that the truncated lozenges are usually named *scuta*, which were the Roman legionnaires' shields.
6. I call "tartan grid" an orthogonal grid in which the distance between two successive parallel lines can take two values.

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About the author

Bernard Parzysz is Professor of mathematics – now emeritus – at the University of Orléans (France). His teaching was essentially devoted to math teachers' training and one of his research topics was – and still is – the teaching and learning of geometry. His interest in geometric mosaics was aroused in November 1994 when a Roman *domus* was excavated not far from his flat in Metz, where he was working at the time; the local newspaper reported the fact and highlighted the find of a mosaic (analysed in § 2 of this article). This led him to study the geometric pattern of this mosaic in a paper published by a local journal. He has been working extensively on this topic since 2004, trying to find out the models implemented by the antique mosaicists and group them into "families", in order to uncover the constructive processes put into play.

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Keywords: N4C joint, grid, lattice, structure, construction technique

Research

The N4C Joint

Abstract. This paper discusses the discovery that four oblique prisms with a square cross section can intersect, forming a joint by means of a notch parallel to the horizontal plane. The use of this joint in new constructions is explored.

Introduction

Whilst drawing, I discovered by chance that four oblique prisms with a square cross section can intersect, forming a joint by means of a notch parallel to the horizontal plane. I am a sculptor, but my love of architecture led me to develop this joint for use in new constructions; having drawn and executed a large number of sculptures, I felt the need to change the direction of my work and direct it towards the 'useful', and the sculptures were what led me in this direction. Reading and studying the principles of analysis of shape, and the philosophical texts on light and space, and on interior and exterior spaces, written by the great architects of the first half of last century, along with my need to build rather than model, led me to bind together modules and spatial networks based on this joint.

The demonstration of this regularity had to obey one of the elemental theorems of plane geometry, that is, a triangle inscribed inside a semi-circumference whose longest side is equal to the diameter, is a right triangle (Euclid's *Elements*, Book IV, Proposition 5). The longest leg of the triangle and the angle it forms with the hypotenuse is the line sought, that is, the inclination of the members forming the joint. But another condition had to be fulfilled to make the joint possible: the space separating the two parallel members had to be equal to the length of the side of the cross section of the member (fig. 1).

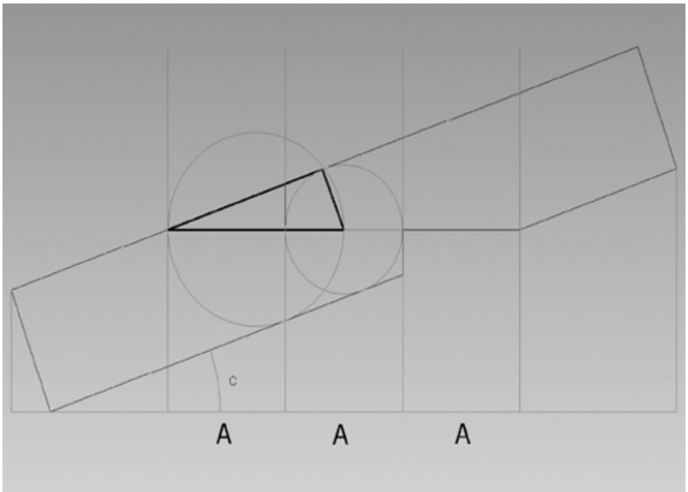

Fig. 1.

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Having discovered the starting point of the oblique line and that the angle it forms with the horizontal is 19°, I then explored the reason for this union. If we look at the projection of one of the members on the vertical plane (in a drawing) and draw a horizontal line which intersects the two oblique lines or edges of the prism, the a-a′ segment which is formed is three times larger than the distance between the two parallels (section of the prism). Dividing this segment into three sections and drawing a perpendicular through these points, two right-angle triangles are formed, and the two legs are the projections of the planes which intersect the prism forming a notch for its assembly; the longest leg is the side of the square and the bearing and contact horizontal plane (fig. 2).

If we now look at the projection of the prisms on the horizontal plane, the horizontal line mentioned previously is the side of a square; this square is equal to a horizontal plane which contains the four bearing surfaces of the four prisms forming the joint, these being separated from each other by another square of the same dimensions (fig. 3).

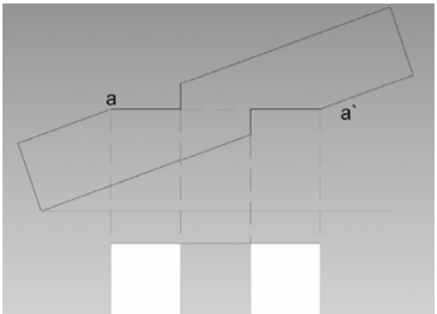

Fig. 2

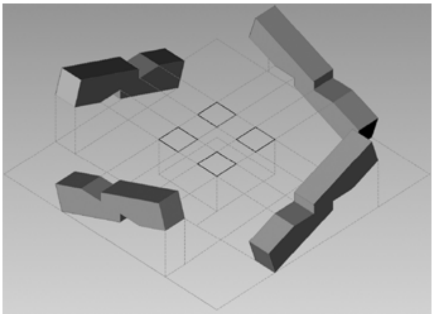
Fig. 3

Once the four bearing surfaces are situated on the same plane, we can repeat these notches all along the prism to form different compositions. If the distance between notches is equal, a three-dimensional space network is formed.

The first piece executed using this joint was “Four squares inscribed within a cube” (fig. 4), a composition formed by two pairs of squares parallel and perpendicular to each other creating two joints aligned on the same vertical line. This sculpture gave the joint its name: N4C, since this closed construction is the smallest one that can be built with two parallel joints; the other pieces are more complex compositions.

One of the complex shapes that can be created using this joint exists in Mandelbrot’s fractal geometry. I call it the “cardoon”, given its similarity to the artichoke thistle in the way it grows (fig. 5). An N4C joint leaves four ends free; if we extend them in order to make another joint we obtain four more joints, if we repeat this operation on each of these new joints, we will create a geometric progression. Of greatest interest in this progression is that the different joints can be joined together using one of their members, creating a structure that is interlinked and has infinite possibilities for growth (fig. 6).

The application of the N4C joint in architecture

In the construction sector, using this joint to connect up to four members with a rectangular section brings to mind prefabricated elements for the rapid installation of a metallic structure, after the fashion of architecture using steel and glass.

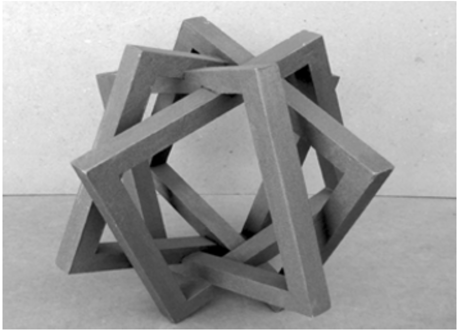

Fig. 4a

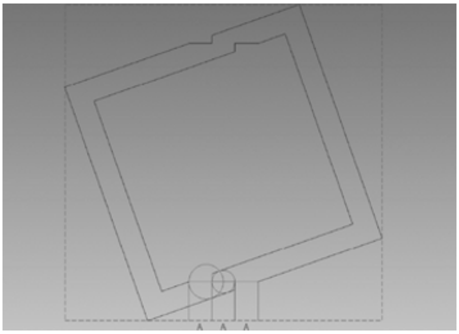

Fig. 4b

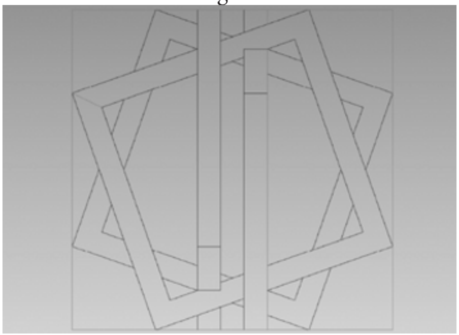

Fig. 4c

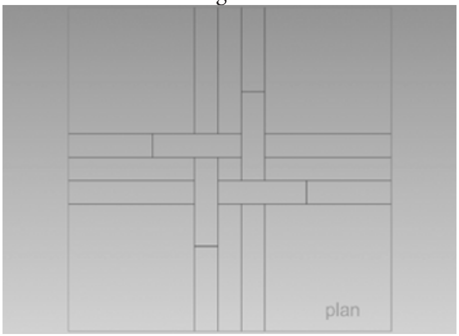

Fig. 4c

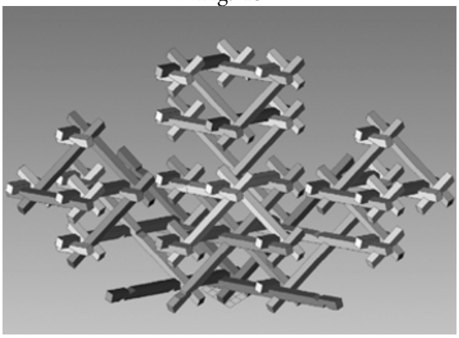

Fig. 5

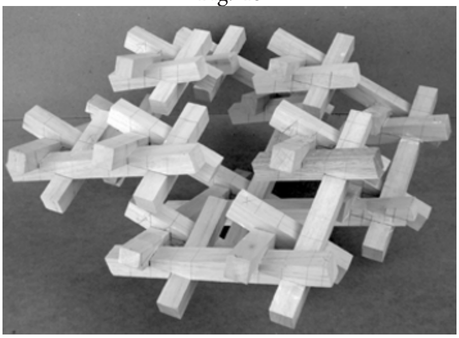

Fig. 6

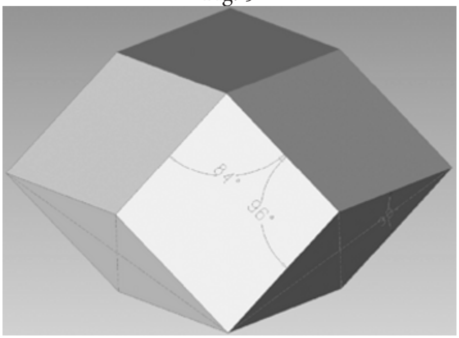

Fig. 7a

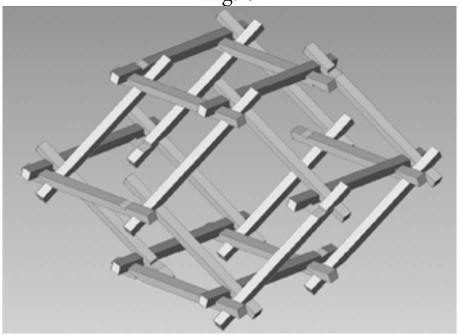

Fig. 7b

The starting point is the rhombic dodecahedron which can be formed using this joint. (fig. 7a,b). Placing the two longest axes of the prism on the horizontal plane, our working plane is the four upper faces (84° and 96° rhombi), which join at their upper vertices, their lower vertices being the bearing points on the horizontal plane, and with a 19° angle. As an alternative, by placing the polyhedron with its two longest axes on the vertical plane, the four aforementioned faces in an oblique position closer to the vertical plane; their angle to the horizontal is 71°, these being supported by one of their vertices on a vertical plane (fig. 8a, b).

If we substitute the arrises of these faces (rhombi) with rectangular prisms with a square cross-section (steel pipe) and we make notches at their ends, we form a frame. If we then make various notches on these four members at equal distances, we create a mesh whose elements are parallel to the sides of the frame (rhombus) (fig. 9).

By taking the four faces of the upper half of the polyhedron in the vertical position, we obtain four frames which are two by two parallel, and the properties of this joint allow any member in one of the planes to be connected to each one of the joints in the other three frames. This can be clearly seen by making a construction similar to the rhombic dodecahedron mentioned above using lengthwise members with a square cross-section (fig.10).

The compositions that result from using the planes of the polyhedron faces in the horizontal position resemble pyramid shaped roofs, the four arrises (beams) joining at the upper vertex in the N4C joint, and supported on the horizontal plane formed by the ground, or completing the entire grid of the face of the rhombic dodecahedron, creating an open structure and giving more floor area (fig. 11a, b).

These four members or arrises must have two notches along their length and at a separation distance equal to that of the crossbeams making the grid. The remaining members merely require a notch on one of their faces to permit intersection.

Using beams with a set number of notches along the length of their faces and the same separation distance between them allows us to alter the surface of the faces and thus the ground plan projection of the roofing formed by the four frames. The relation between the section of the beam and the separation of the notches, and the length, provide us with the formula for calculating the strength of the entire structure.

This joint makes it possible to create compositions with a square ground plan, joining several equal constructions together, rotating them and leaving spaces between them (fig. 12).

In the same way, by using roofing made of beams of different lengths they may be interconnected, thus increasing useable floor space (fig. 13).

Finally, if the stresses acting on the structure make it necessary, this joint makes it possible to reinforce its interior area, strengthening the wall where it joins the horizontal plane of the ground, through the connection of the four members that make up the joint, while at the same time leaving greater height in the access points to the interior (fig. 14). We can also brace the upper vertex of the pyramid on the inside by using a structure that is the inverse of the one built.

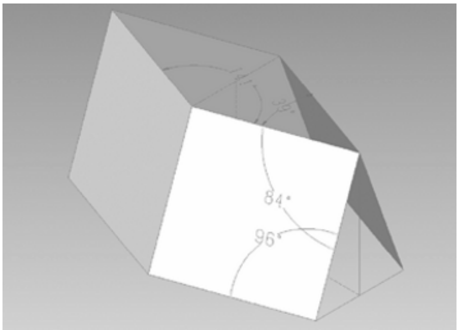

Fig. 8a

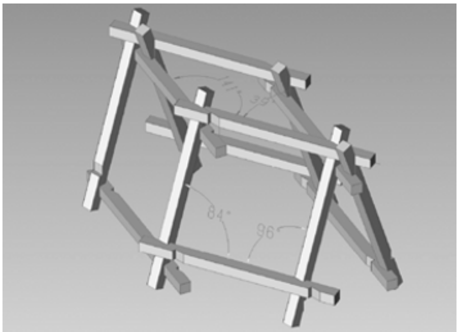

Fig. 8b

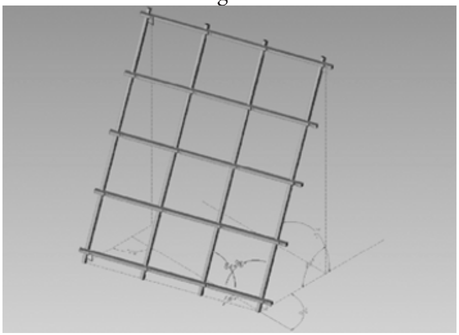
Fig. 9

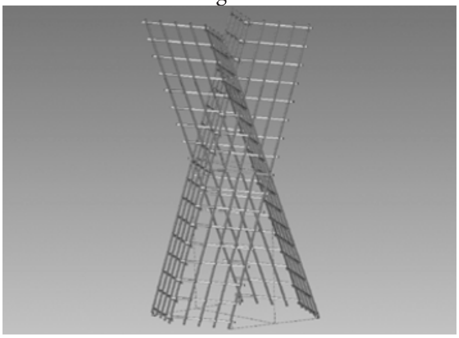
Fig. 10

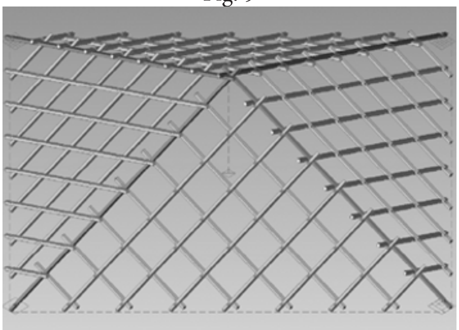
Fig. 11a

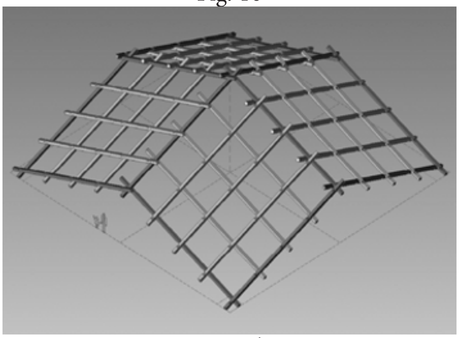
Fig. 11b

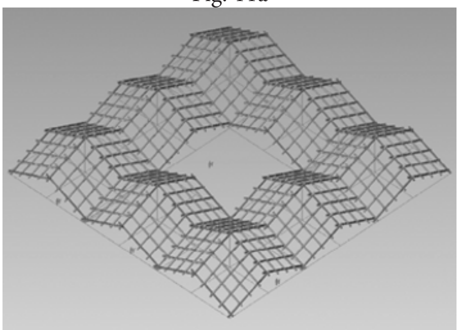
Fig. 12

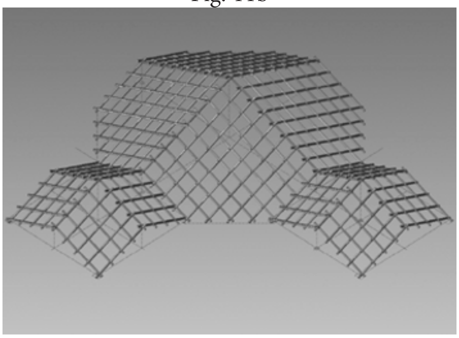
Fig. 13

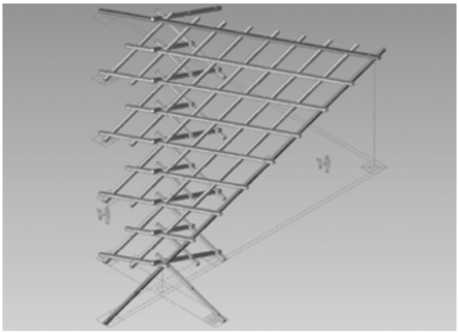

Fig. 14

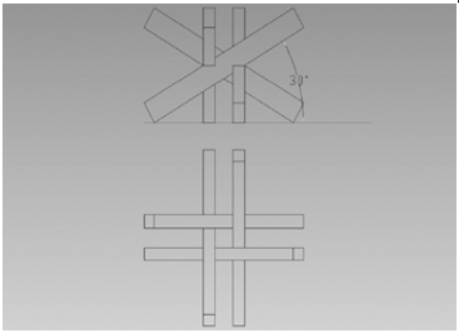

Fig. 15a

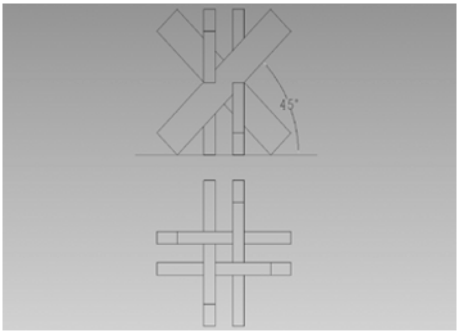

Fig. 15b

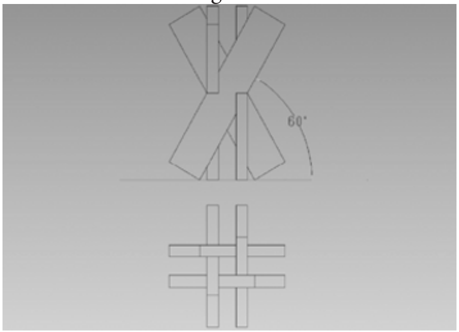

Fig. 15c

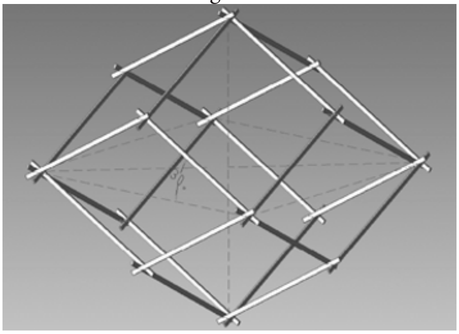

Fig. 16

Fig. 17

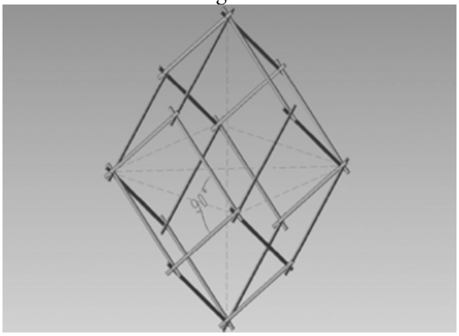

Fig. 18

N4C with different angles of inclination

The essential pre-condition required for the joint is that the four bearing surfaces be on the same horizontal plane. Increasing the angle of inclination makes the horizontal section of the prism smaller, so its width must be increased to allow room for the notches on the one hand, and on the other the separation distance between them must be sufficient from the perspective of beam strength.

Fig. 15 shows three examples of this joint with prism inclinations to the horizontal plane of 30°, 45° and 60°.

For this reason, the section of the prism must be rectangular, with the longer side positioned vertically and the shorter side supported by the other prism. In the case of the 30° and 60° angles, the polyhedron formed is still a rhombic dodecahedron (figs. 16, 17) while in the case of the 45° angle the polyhedron has eight rhombic faces and four squares (fig. 18).

Building with these angles still results in pyramid shapes whose upper vertex becomes more acute as we increase the angle (figs. 19, 20, 21). One of the properties of this joint for configuring different compositions is that the notches of the four members are on the same horizontal plane (fig. 22), which means that a number of these constructions can be joined together on the horizontal plane (see fig. 12).

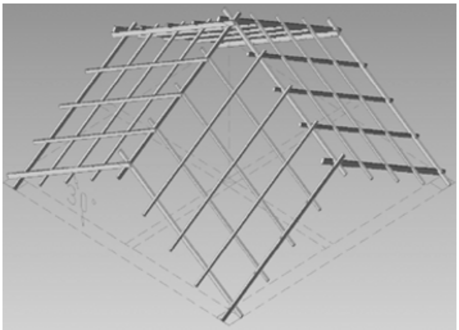

Fig. 19

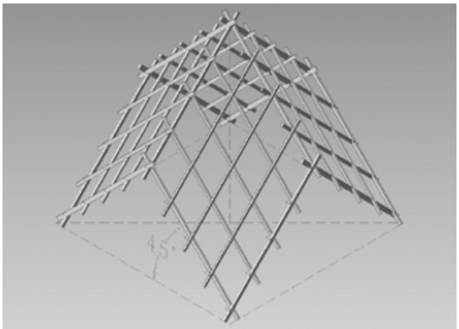

Fig. 20

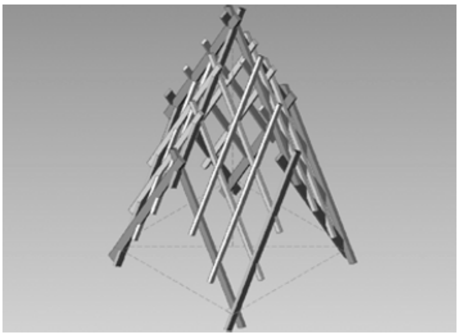

Fig. 21

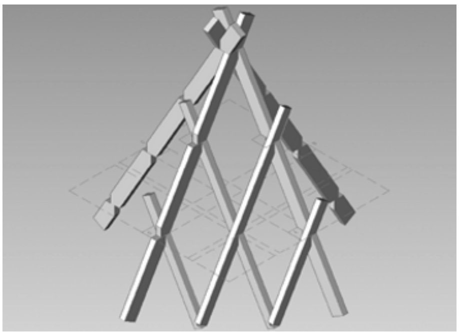

Fig. 22

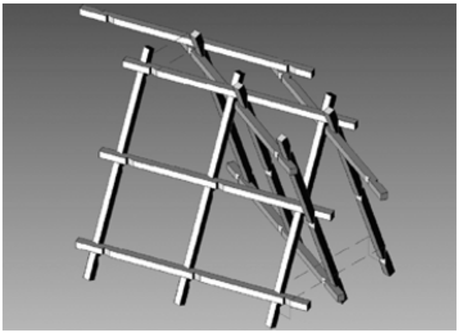
Fig. 23

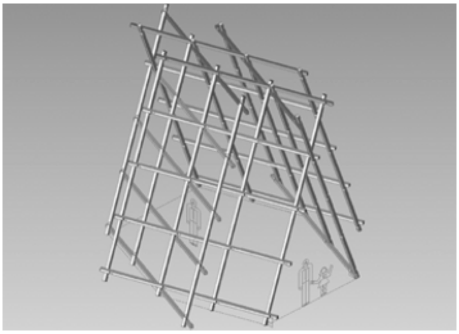
Fig. 24

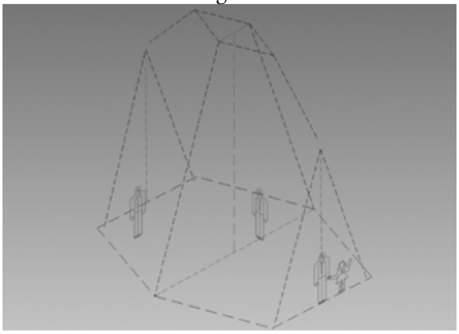
Fig. 25

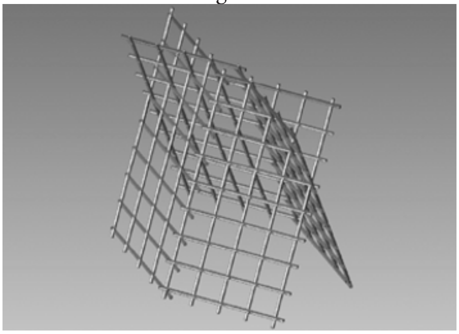
Fig. 26

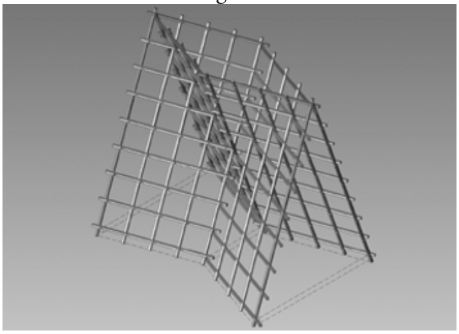
Fig. 27

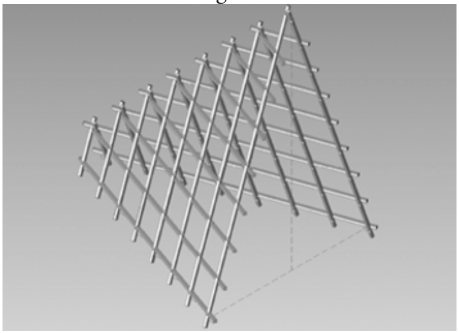
Fig. 28

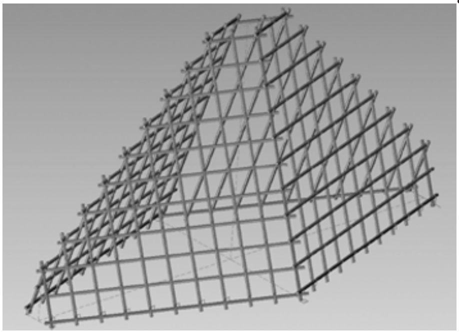
Fig. 29

Taking the working area to be the four faces of the upper half of the polyhedron in the vertical position, the planes offer greater slenderness as their angle of inclination to the horizontal plane is 71°, even though the characteristics of the joints between them are very different. Their arrises intersect, but this is achieved by means of the N4C joint which enables the four members to intersect (fig. 23).

Substituting the arrises of these faces of the polyhedron, forming a frame and creating a structure as in the previous example (the same flat structures, but rotated), four planes which are two by two parallel are formed, which we can imitate and intersect (fig. 24). Let us examine the upper arris of each of the planes: we make them intersect at one point and at the same distance from their ends, the members forming the structure always intersect where the notches which allow intersection and formation of the joint are located. The interior space created by this construction differs from the previous example; its height is three times that of its ground plan projection; and an atypical 'vault' is formed by the junctions or joints where the four planes intersect (fig. 25).

As in the example given previously, using beams of different lengths but with equally spaced notches results in a wide variety of compositions that can be executed.

I call these dihedral plane compositions "vertical architecture", to differentiate them from pyramidal roofs. A greater number of compositions can be created with these dihedral planes. Some examples are: according to the chosen line of intersection between two of its planes (figs. 26, 27); interrupting the continuity of the member at the joining line (figs. 28, 29); positioning the dihedral planes with the concave space on the inside (fig. 30); with this space on the outside (fig. 31); significantly changing the shape of its ground plan (figs. 32, 33). It is clear that the edges of two consecutive dihedral planes cannot be joined by means of the N4C joint (fig. 34a). But if we examine the ground plan projection of the complete joint, the shortest member at one of the ends of the structure is perpendicular to the horizontal member of the other structure located on the perpendicular side of the ground plan. If we extend the two members until they meet, forming an 'L' shape, assembly of the two structures is complete (fig. 34b).

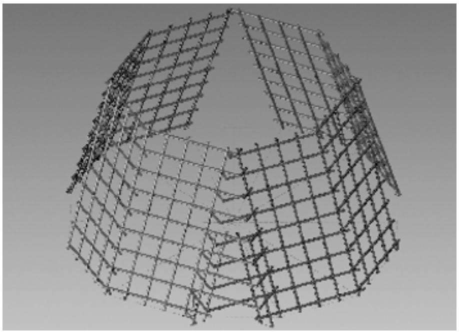

Fig. 30

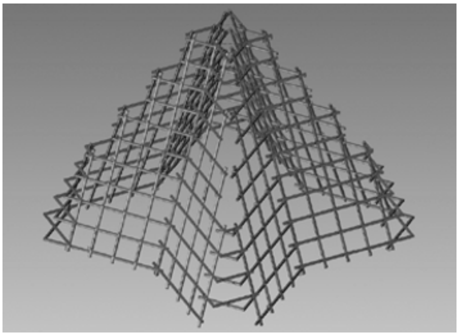

Fig. 31

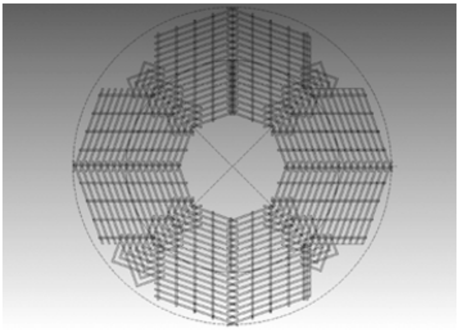
Fig. 32

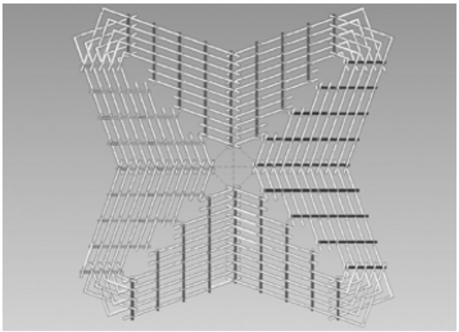
Fig. 33

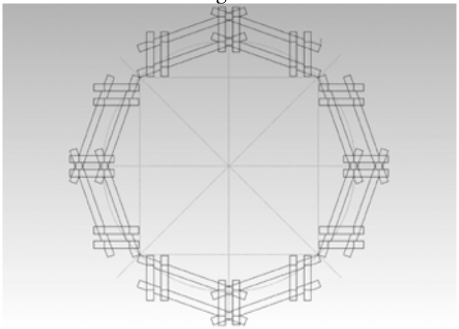
Fig. 34a

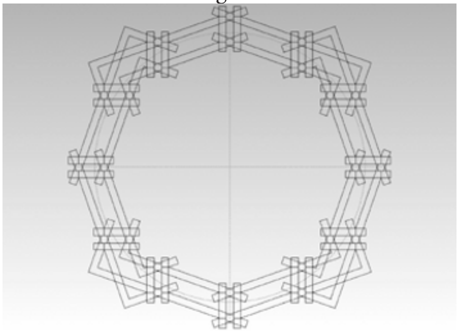
Fig. 34b

Joining four dihedral planes at their non-parallel edges

By placing the rhombic dodecahedron with its two longest axes on the vertical plane, and dividing it into two halves on the same plane, we obtain two equal structures (fig. 35).

If we transpose this division onto our construction with bars joined together using N4C joints, the result is a convex structure whose openings are rhombi with angles equal to the faces of the rhombic dodecahedron, 96° and 84° (fig. 36).

Joining together all the faces using the L-shaped member interlinks the entire construction (fig. 37). If we examine the joints in the upper part of the resulting construction we can see that only three bars intersect, continuing the construction by sliding the same structure vertically, the vertical bars of the upper structure will be those that fill the empty space in the joint. Superimposing these structures creates a tower of infinite size.

Grids

Making a joint with bars which have notches at both ends, and taking two of their parallel elements, we extend them in linear formation, repeating the same operation with the other two elements of the joint (fig. 38). If we keep repeating this assembly we obtain a flat grid which we will position in the vertical plane by rotate through 90° (fig. 39). If we join two of these grids at one of their faces so that the members forming the joint match in a straight line, we create a new grid or a double grid, whose members are fused into a single one; this gives the structure greater rigidity and stability (fig. 40).

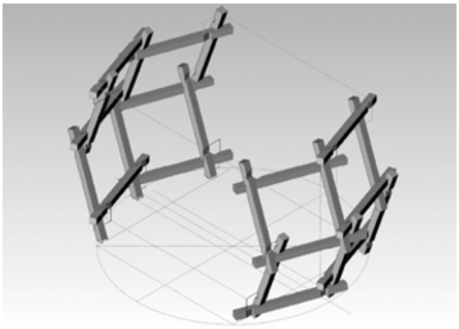
Fig. 35

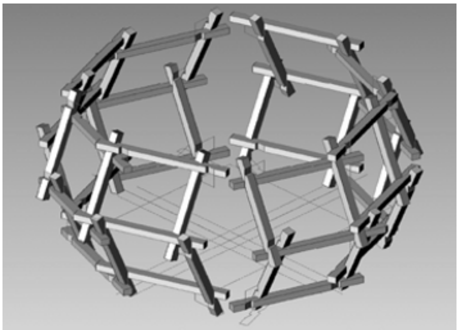
Fig. 36

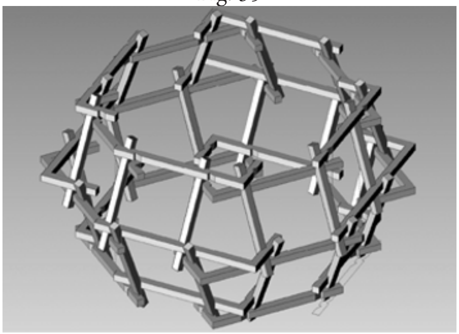
Fig. 37

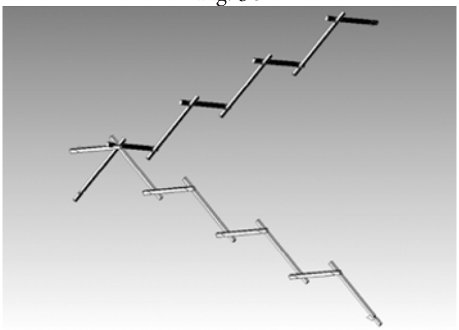
Fig. 38

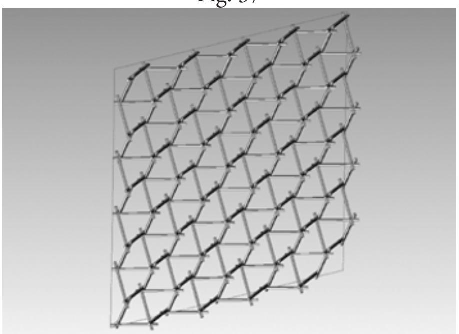
Fig. 39

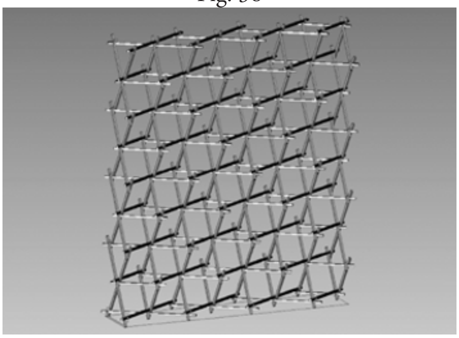
Fig. 40

Joining two of these vertical structures at a right angle can be accomplished in two ways. One way is to position both structures such that their horizontal members are located on the same horizontal plane, then taking two members symmetrical with the bisecting plane and joining them together with a tangent curve, into a single member (fig. 41); this interlinks the two structures. The exception to this is where the joint is 45°, in which case this member will be straight. The other way is by extending a member of each of the structures until they meet at a right angle in a single member (as in fig. 32). Where the grid is a double grid, the two arms of the L will be extended until reaching one or two joints in each grid (fig. 42a, b).

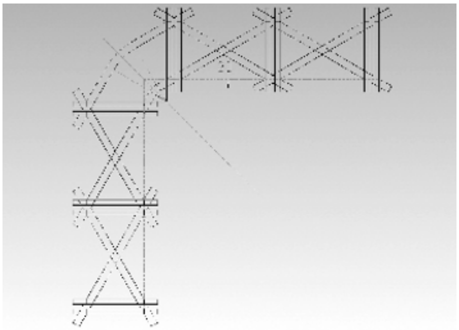

Fig. 41

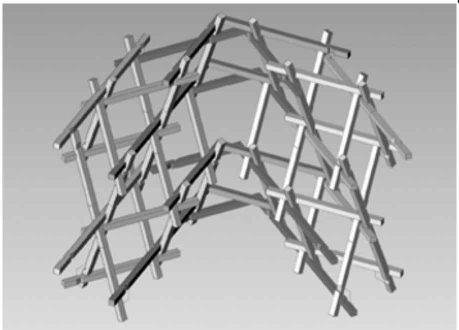

Fig. 42a

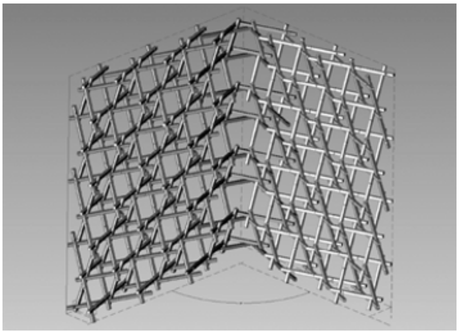

Fig. 42b

Grid on an inclined plane

By making a module with the members of a 30° joint as shown in fig. 43, a grid is created on an inclined plane whose angle is the same as that formed between the central member and the horizontal plane. We create one of these grids in such a way that the starting point is a module; from this first one, two are formed, and from these two three are formed, and so on, creating a triangular structure (fig. 44). This structure is one of the four faces of a pyramid and we can join them together without additional members. When the four faces are joined together, it can be seen that the members making up the arris of this inverted pyramid are one member of each one of the faces fused into a single member, joined together at the notches on one of their ends. This construction can be supported by a pillar whose upper part has a cross section equal to the gap between the members; given its morphology I call it a "tree" (fig. 45).

In the same fashion as these four walls have been joined together, so a number of these trees can be joined together at the upper edge forming a ground plan grid, each supported by a central base acting as a column. In this way, the interior space that is created is similar to pseudo-vaulting formed by intersecting oblique planes (fig. 46).

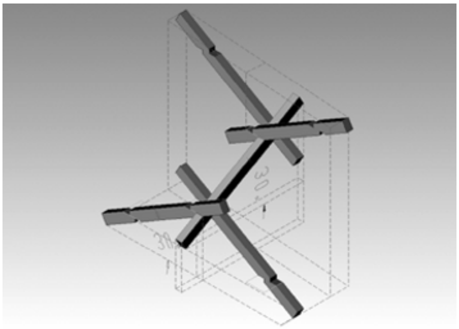

Fig. 43

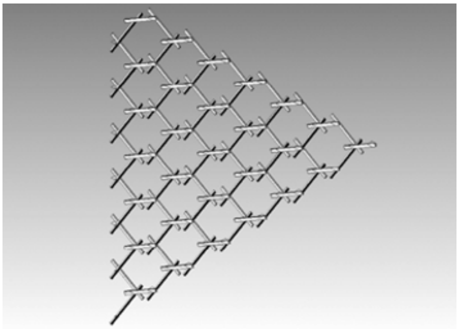

Fig. 44

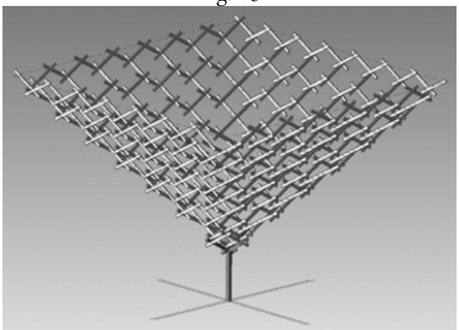

Fig. 45

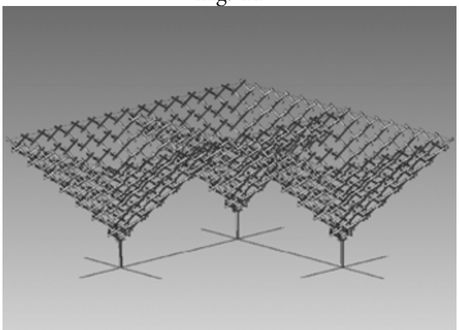

Fig. 46

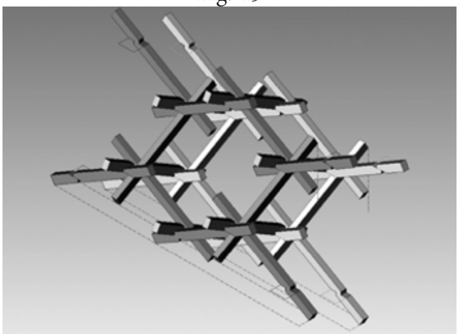

Fig. 47

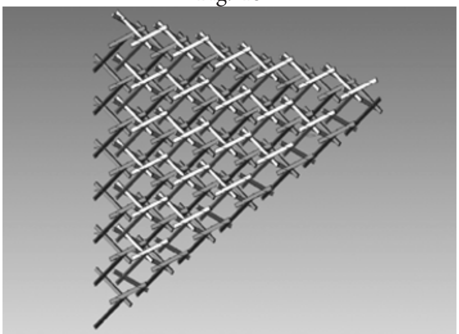

Fig. 48

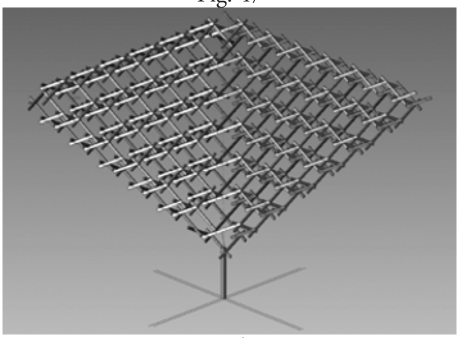

Fig. 49a

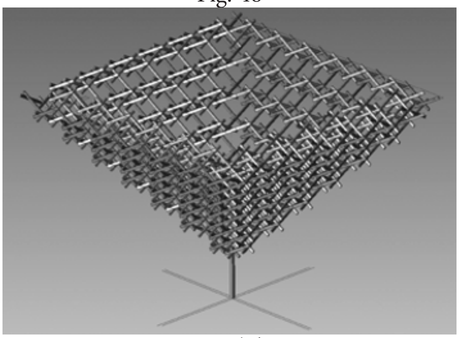

Fig. 49b

Double grid

If we examine the starting module, we can see that the joint is made up of only three members; if we add another member, the free end connects to a new module, creating a double grid following two parallel planes, and thus the structure is reinforced (fig. 47). If we transpose this onto the walls of our tree (fig. 48), these new structures can be joined together in the same way as in the previous case, creating a compact structure (fig. 49a, b).

If we return to the construction shown in fig. 11a and 11b, and examine the joint of the members forming the face of the pyramid, we see that only two members intersect, whereas the joint enables four to intersect. In the case of fig. 11b, positioning one face on top of another following two parallel planes so that their joints are on the same vertical line; these faces can be joined using two shorter members (fig. 50), depending on the separation that we wish to have, thus completing each of the joints with four members (fig. 51). In this example I have used a length equal to the distance between two joints on one of the beams with notches at both ends. If, in addition, we extend the beams from the arris meeting point of the two lower faces, a distance equal to the aforementioned shorter member, until they meet and connect to the upper face in one of their joints, we thus create the most significant linking together of the structure, since all the beams that intersect in the arris, establishing a bearing point, must extend only a very short distance to reach the upper face, thus creating a structure capable of being self-supporting (fig. 52). In the case of fig. 11a, all the members of the upper face will be extended until they reach the horizontal plane of the ground (fig. 53).

Grids formed with intersecting planes

Building flat structures, and considering these as planes which are independent from each other, we can make them intersect in such a way that each of the four members forming the joint belongs to a different plane, the intersecting line being a series of joints in a straight line. To understand this better, we can start construction by joining a member from each one of the planes forming a static joint (fig. 54), then we slide one member on top of another in succession, positioning them in each of the grooves until a lattice is formed (fig. 55). The outcome is the same as would be obtained if four planes intersected, but here it is only possible because of the geometric properties of the N4C joint, the most significant of these being that all the joints of the different members are on the same horizontal plane (the same as shown in fig. 22). The compositions can be more or less complex, depending on the position of the intersecting line in the plane and the number of elements contained in the plane lattice (fig. 56).

Horizontal or vertical linear structures

A linear composition or construction is similar to that of a beam triangulated with pipes. If we construct the four faces of a rectangular prism with members from three joints, one in the centre and two at the ends, and insert into the gap inside the joint a bar of equal cross-sectional dimensions, considered the arris of the prism itself, then where two members of each face of the prism intersect, the four faces are thus interlinked. By forming a core with four members from three joints, the members joined by the central joint and the joint at their ends linked to the prism faces at the central line parallel to the arrises, we thus ensure that the entire assembly is adequately braced (fig. 57). Fig. 58 shows the linear continuity, proving the efficacy of the core joined to the walls.

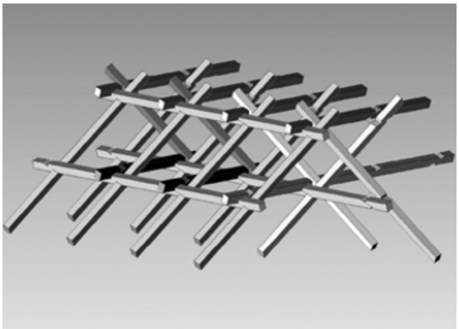

Fig. 50

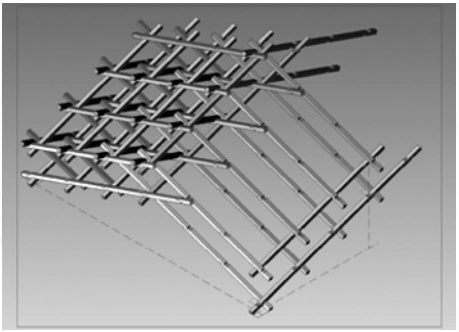

Fig. 51

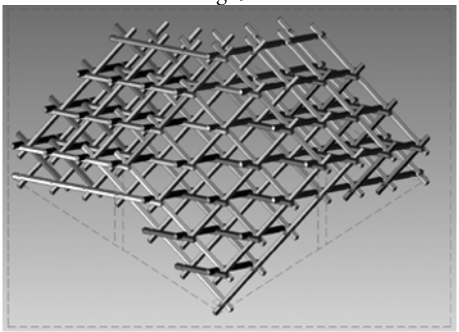

Fig. 52

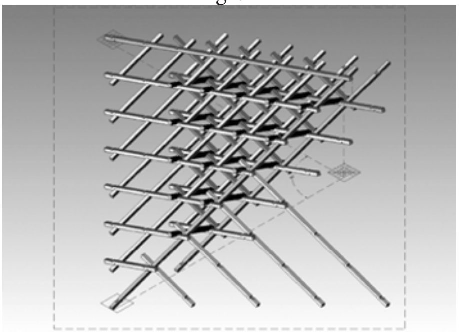

Fig. 53

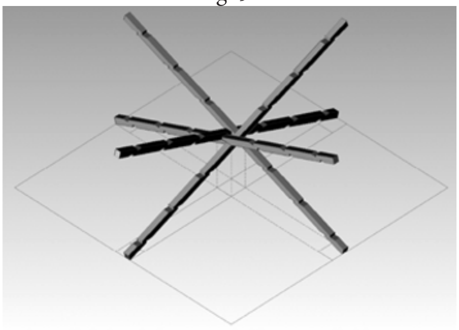

Fig. 54

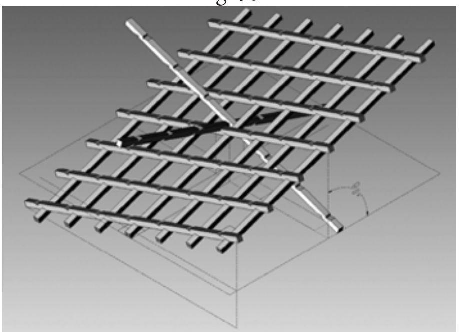

Fig. 55

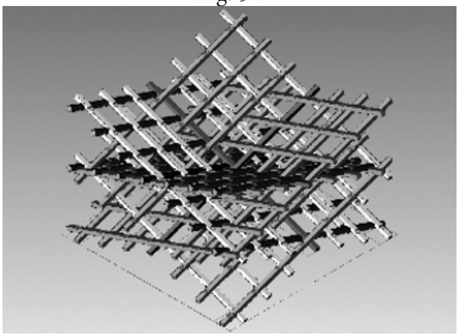

Fig. 56

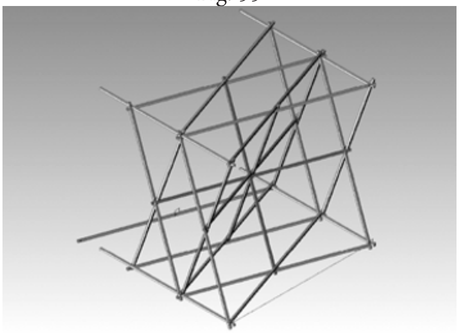

Fig. 57

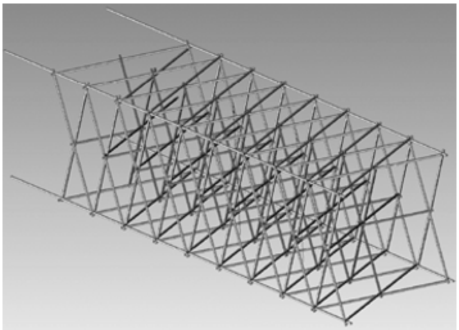

Fig. 58

Conclusion

The main reason I consider the discovery of this joint important is because of its geometric and structural properties; I say geometric, because the conditions it imposes for spanning space are always fulfilled.

As a result of these properties, the structures that can be executed have that idiosyncrasy of shape which makes them new, if we bear in mind that they do not require additional members.

Acknowledgment

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About the author

Jesús Molina (Spain 1949) is an autodidact. He started his studies to become an Architectural Technician but he did not finish them in order to travel around Europe. In Venice he attended the International University of Art with Professor De Luigi. He stayed at Pulteney College in London. He was awarded the First National Design Prize for the design of a sculptural module with didactic applications in geometry. This work was exhibited in the School of Architecture in Madrid. His work titled "The Public Space" was shown in the Town Planning Course of the Menéndez Pelayo International University, UIMP. His sculptural works are present in several Spanish cities. His work can be seen at http://www.esculturajesusmolina.com.

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Didactics

Geometries of Imaginary Space: Architectural Developments of the Ideas of M. C. Escher and Buckminster Fuller

Abstract. The aim of this paper is to show how efficient mathematical models can be in simulating situations that are apparently distant from one another. The works of M. C. Escher and Buckminster Fuller are used as starting points for the exploration of the first applications of non-Euclidean geometry to architecture. The investigation goes on to fractals and chaos, nurbs, blob architecture and deconstructivism.

Keywords: M. C. Escher, Buckminster Fuller, H.M. S. Coxeter, Benoit Mandelbrot, chaos theory, fractals, nurbs, blob architecture, deconstructivism

Introduction

A great variety of examples confirms the relationship between mathematics and architecture. The aim of this paper is to show how efficient mathematical models can be in simulating situations that are apparently distant from one another.

Some questions immediately arise concerning the teaching of architecture and mathematics in architecture schools. Which is the appropriate role of mathematics in architecture? Which is the architectural use of mathematics? "What" must students know nowadays? Mathematics is a fundamental calculation tool for the structural planner; but the mathematical foundations embedded in computer science are also important, since these govern design and representation, thanks to the ability of computers to make it possible to visualize virtual buildings.

Mathematics sometimes appears in architecture itself. The reference to geometry as a design tool is evident in many examples, such as, for instance, when a search for numerical and geometrical programs is applied to the organization of rooms in a design.

In order to answer these questions, it is necessary to estimate "how" and "if" mathematics plays an active role in the design and consequently, if it can be considered a dynamic instrument in the design process. Therefore, starting from Euclidean geometry and its derivations, it is necessary to verify the actual existence of relationships between mathematical research and architectural research in order to understand if the former has influenced the latter; and finally if progress in modern mathematics can still produce innovative formal results.

When mathematical models are constructible, they become real architecture, but even when they are not constructible, they still inspire architects. For a long time architects were satisfied with elementary shapes, and used forms and surfaces generated from the

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movement of relatively simple curves, such as ellipses and spirals. Their genius mainly lay in realizing buildings of great complexity and fascination based on very simple models. But today this simplicity seems to have lost its power to stimulate.

If mathematics suggests ideas for architecture, then modern research should trigger the interest of the architects, not only because of their possible applications in computer graphics – which remains a mediating instrument – but mostly because of the possibility to develop innovative building shapes.

How did the "other" geometries, posterior to Euclidean geometry, influence architecture? Are the new modern mathematical studies destined to leave a sign in modern buildings? Can architecture be inspired by non-Euclidean geometry, in its material being, which remains very distant from the virtual one?

In some recent buildings, computer-aided design makes use of curved surfaces having particular properties, such as nurbs, or it develops shapes from fractal models, which take advantage of the random repetition of simple algorithms. These phenomena may take place without the architect being directly aware of the mathematical science behind it.

Thus there is a close relationship between *form* and *formula*.

The central question is always the actual materiality of architecture, which needs to be effectively built and which is linked to the reality of three-dimensional Cartesian space. The fourth dimension of "time" appears in the rhythmic partitions that link architecture to music, but it remains rather marginal, because architecture is generally meant to be "immovable" and "eternal". It is even more difficult to design buildings in a *n*-dimensional space, as those suggested by some post-Euclidean geometries.

Mathematicians, whose constructions rely on logic and not on statics, have a greater degree of freedom than architects. But while they can freely follow their imagination, they sometimes need the help of drawings in order to explain their forms. In fact, drawing plays a determining role in the relation between geometry and the art of building, thanks to its ability to reduce the three-dimensional essence of the architectural space to the two-dimensional sheet of paper. This emphasizes the dichotomy between the *idea* and its *representation*, thus between *imagination* and *image*. However, drawing is sometimes also able to illustrate the underlying *mental space*.

Few mathematicians and architects interact beyond expressing a mutual but generic interest for their different skills. In contrast, we chose to organize this research as a common work, confronting diverse disciplinary specializations and skills, and occasional misunderstandings of technical language. The first step was the preliminary collection of meaningful works of architecture, and their evaluation on a mathematical basis. The goal of the research is to investigate the possibility of creating new shapes from modern mathematical concepts.

Escher's works

The works of some artists generically related to Expressionism, such as M. C. Escher (1898-1972) [Escher 1982] and Buckminster Fuller (1895-1983) [Gorman 2005], are an interesting starting point for the exploration of the first applications of non-Euclidean geometry to architecture. Starting in the first decades of the twentieth century, both Escher and Fuller based their studies on mathematics, and thanks to their acquaintance with some mathematicians, their creations anticipated some contemporary architectural research.

The lifelong research of M.C. Escher inquires into the “geometrical structures” of space, producing complex constructions that stress the ambiguities between the drawing and the projective space. His work starts from the study of regular grids, as a main reference and measuring tool of infinite space. Although Escher was not an architect in the strict sense of the word, but rather a graphic artist, he gave a visible image to the relationships between mathematics and architecture. Through geometry, his drawings investigate mental constructions and imaginary worlds, raising tricky questions about the structure of physical reality and its sensorial perception. These graphic constructions transport us to a world that seems to be physically impossible, but which is as logical as the laws of mathematics.

Speaking about his own work, Escher said that it was beautiful and ugly at the same time: beautiful, because the drawings, which are the result of research that goes far beyond its first achievements, are astonishing; ugly, because it remains unresolved, lying in the ambiguous realm between physical space and perception, in an obsessive game between the two dimensions of the drawing, the three dimensions of the Euclidean space, and the many of the sensorial space-time. “I have played a lot of tricks, and I have had a fine old time expressing concepts in visual terms, with no other aim than to find way of putting them onto paper. All I am doing in my prints is to offer a report of my discoveries” [Ernst 1995: 14].

Escher’s studies of symmetry recall Semper’s theories on the genesis of architectural and industrial design shapes [Semper 1860-63], and Gombricht’s analyses on the concept of order [Gombricht 1979]. Escher searched for mathematical perfection in a chaotic universe; in his acceptance speech for the 1964 Hilversum prize, he testified “that we live in a beautiful and ordered world and not in a chaos, without rules, as it can seem” [Escher 1982: 124].

Escher thought that the laws of mathematics and geometry are not constructions of the human mind, but “*they are*”, as the crystals “*exist*” in their perfect shapes. Therefore, the beauty, order and symmetry of Plato’s regular solids, together with the creations of nature, become unavoidable models.

His whole work illustrates an imaginary world, which is built up on the logical bases of geometry, drawn both in plan and in space, thanks to the repetitive use of grids and symmetry. He gives a “visible” shape to some abstract concepts suggested by mathematics, such as the “infinite”, the dichotomy between unity and continuity of the numerical sequence, and space-time relativity. He emphasizes the paradoxes of perception in representation, playing with the ambiguity of the projective systems. Thus, Escher illustrates his world thanks to the regular structures of the drawing; his study is based on Euclidean geometry, but he does not hesitate to follow other paths to express the paradoxes of imagination. He creates a visualization of abstract concepts coming from the studies of the mathematicians and the crystallographers, with whom he establishes a fortunate collaboration. The use of modular grids completes his systematically regular division of the plan and extends to the three-dimensional symmetries in the space. The symmetries of regular polyhedra had already triggered Plato’s and Leonardo’s interest before him, but Escher goes beyond the initial postulates, and pushes the formal logic to its extreme consequences. In a consistent manner, Escher follows the road that the mathematicians show him, going beyond the familiar world of Euclidean geometry and codified drawing techniques.

The leitmotiv of those graphical constructions of imaginary spaces is the striving to actually draw the *infinite*. This topic comprises both the concepts of *unity* and *variety*. So its representation becomes a cosmogony, giving an image of the created one: "We find it impossible to imagine that somewhere beyond the furthest stars of the night sky that there should come an end to space, a frontier beyond which there is nothing more [Ernst 1995: 102].

What is larger than the infinite? Is it possible to simulate it in a model?

In past centuries, architects and artists tried to represent the infinite depth of space. The interior space of a Gothic cathedral was intended to give the feeling of an endless room. Later, Renaissance painters found a different answer to the problem thanks to perspective, which was able to simulate the infinite depth of space on the sheet of paper or canvas. Here mathematics seem to follow painting. Some time later, the study of conic sections, together with projective geometry and the codification of descriptive geometry – based on the representation of the improper point – produced the spectacular Baroque architecture. This is also the time when *quadratura*, illusionistic painting usually applied to ceilings, and *anamorphosis* appear, in which it becomes difficult to distinguish the actual space from the perceived space.

In a way that is similar to the cathedrals builders and the Renaissance painters, but with a different approach, Escher succeeds in "building the infinite" by means of the reproduction of cyclical divisions of the plan applied to the surface of the sphere. By doing so, he eliminates the actual difficulty of repeating a same pattern on a plane surface ad infinitum, and the continuity of an endless drawing is achieved on a closed, continuous surface.

The cyclical tessellation, inspired by the geometry of arabesque decoration, was the starting point of Escher's research. The regular division of the plan recalls the principles defined in the late-nineteenth-century by the grammars of ornament. In antiquity, those were also applied to the patterns used in wall painting. Escher extends the model to the pure concepts of *surface* and *space*, adapting his drawing patterns to the spherical continuous surface. Thanks to the multiple symmetries of the five regular solids, he is able to construct drawings without interruptions: the surface of each face is finite, but the whole solid surface and the variety of its symmetries is endless. It would be impossible to realise a surface of limitless extension on the plane. Circumscribing a spherical surface, the symmetries of the polygonal faces of the regular polyhedra produce spatial symmetries. However, its projection on the plane forms radial grids, like those of rose windows and floor patterns. Therefore, radial patterns can be read as projective transformations of three-dimensional shapes, such as domes, emphasizing the ambiguity between design and drawing.

While perception may be an illusion, the rigorous logic of mathematics explains the geometrical structure of the space. So the image of the "infinite" appears in the continuous growth of spirals, in Moebius strips, and in the drake biting its own tail.

In Escher's creations, some contemporary studies regarding perception and the relation between *figure* and *background* are already present, visible in the regular black and white divisions of the plane. The graphical oppositions and the contrast between opposites become a pretext for revealing the total fusion of antithetical concepts: day and night, angel and devil, but also over and under, inside and outside. These last are expressed in a sophisticated way in his perspective studies, where he shows the illusion of the perceptive space, through the logical rationality of the paradox. Contradictions in the drawing reveal the ambiguity between the feeling of the spatial structures and their infinite depth.

Escher is not satisfied by the compromise of Renaissance linear perspective, which comes from a simplified concept of vision. His ambiguous applications of perspective in the depiction of impossible architectures testify to his dissatisfaction with the rules of codified drawing. His perspective studies attempt to follow the physiology of vision, the mobility of the eye and the relativity of time to perception. His curved spaces are far from Panofsky's theories about the perspective knowledge in antiquity [Panofsky 1997], but it is quite easy to establish a link between ancient perspective and Escher's projective constructions.

In fact, Euclidean geometries were not sufficient for his research. The compositions related to Poincaré's drawing of hyperbolic space (*Circle limit I, II, III, IV*) show a relationship to mathematical studies about "other" geometries, and the symmetries of polytopes. Escher is perhaps the first artist to be interested in the abstract concepts of contemporary mathematics, and he is able to illustrate these concepts, pointing the way to more recent architectural research. He gives representational shape to the structures of some non-Euclidean geometries, such as in the grid deformations of *Print Gallery* (related to a Riehman surface with its empty centre). But he also anticipates some later concepts, like those of Mandelbrot's fractals, in *Square Limit*, designed in 1964.

Fuller's works

Escher's graphical inventions are reflected in the works of architect-engineer Buckminster Fuller. Fuller studied the spatial symmetries of the regular polyhedra in order to solve structural problems, and he applied this geometry to the design of light, prefabricated and transportable structures. A mathematical program, derived from the geodetic divisions of the sphere and/or the multiple symmetries of the polytopes, generated both an architectural shape and an actual construction. He then applied the same model to the problem of representing the earth, coming up with an innovative cartographic projection, especially considered for use in air navigation.

Fuller's domes offer a good field of application for testing the architectural potential of Escher's inventions and suggestions inspired by mathematicians.

H. M. S. Coxeter (1907-2003), a friend of both Escher and Fuller, is the key figure for the comprehension of the recent mathematical interpretations of architecture based on topology and chaos theory. The starting point of Coxeter's research was the symmetrical structures of the kaleidoscope, from which he developed the study of the polytopes. These are complex objects in *n*-dimensional space; they do not exist in the real world, but they can be mathematically described. They inspired Escher's work on *Circle limits*.[1] Fuller's domes are also inspired by Coxeter's studies of space symmetry, which would lead to the discovery of a new shape of carbon crystallization. "Buckyballs" are empty spheres with 60 carbon atoms on their surface which form a grid of hexagons and pentagons, like in Fuller's geodesic domes, for which they are named.[2]

Non-Euclidean geometries and architecture: meaningful examples

Chaos theory and fractal geometry, which inspire deconstructive architecture, are in contrast with these tidy symmetries. In fact, deconstructive architecture abandons orthogonal cages, and allows the volumes to break out, creating apparently unordered organic shapes, such as those of blob architecture.

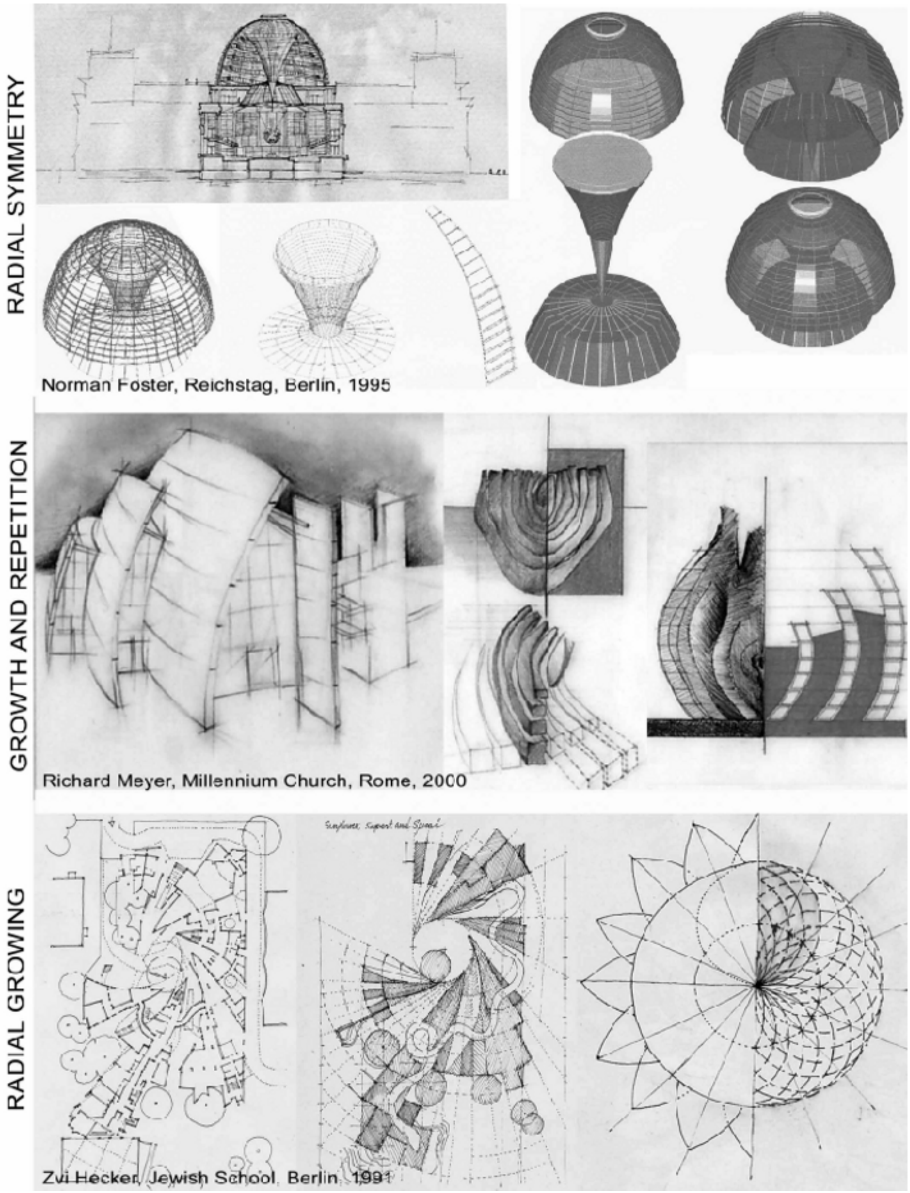

Fig. 1.

Fractal geometry, defined by Polish mathematician Benoît B. Mandelbrot (1924 –) describes some objects that are generated by using repetitive algorithms, reproducing self-similar shapes like the branching of trees, ferns and cauliflowers. From these models both mathematicians and architects have drawn design inspiration.

But repetitive principles also appear in the symmetries of plane tessellation, or in the growing construction of the spirals. Self-similar structures can also be found in the design of the Indian *kolam* and in Celtic knots. In architecture, this concept appears in the spiral dome of S. Ivo alla Sapienza in Rome by Borromini (1642), in the repetition of octagonal shapes in Castel del Monte in southern Italy (mid-thirteenth century), in the elliptic domes of San Carlino alle Quattro Fontane in Rome by Borromini (1634), or in the network of arches that forms the dome of the Chapel of the Holy Shroud in Turin by Guarini (1666), among many other examples.

Finally, there is a reference to fractals in many works of modern architecture: in the self-similar triangles of Palmer House by Frank Lloyd Wright (1950-51) [Eaton 1998]; in the spherical shells of the Opera House in Sidney by Jørn Oberg Utzon (1957-73); in the sails of the Chiesa della Divina Misericordia by Richard Meyer in Rome (2003); and in some works by Zvi Hecker, like the Jewish school of Berlin (1991), the spiral apartment house in Ramat Gan (1981-1989), the polygons of the Hebrew University in Jerusalem (fig. 1).

The mathematical research into "chaotic order" helps us to understand the articulated shapes of "Deconstructivism",[3] including the recent Bird's Nest stadium in Beijing by Herzog and De Meuron (2008). These complex works of architecture, with surfaces that are bent or broken, achieve a controlled disorder through reiterated permutations of regular or simple shapes, which are difficult to conceive without computer elaboration.

Thanks to a controlled randomness, the computer introduces a chaotic element into the architecture layout, applying models that imitate the contemporary world, where it is difficult to recognize a classical concept of order.

Architecture evokes the continuous stress between the tidy cosmos of Creation and the chaos that preceded it, with surfaces which suggest continuous transformations. In these works of architecture, inspired by the earliest studies in topology, spaces no longer obey Cartesian rigor, but imply a continuous formal evolution. The walls are tilted and open: the concept of enclosure is broken by transformations that follow topological criteria (according to which a cube can become a sphere). Bending and distortions are achieved through a succession of shape deformations described by non-uniform rational B-spline (nurbs) surfaces. A meaningful example is the Prototype House in Long Island by Greg Lynn, composed by three altered volumes.

More recently, architectural design was inspired by the organic shapes of blob architecture, of which Greg Lynn is the main exponent. His work contrasts the natural laws of static with dynamic and interactive shapes, generated from the simulation of customers flows, like in the St. Gallen Kunstmuseum (2001). Blob architecture is the result of the application of deforming forces on initial shapes, emphasizing the elasticity of the volumes.

The role of the tool used for representation in the design process is fundamental, since the software programs commonly used in architectural design are based on geometrical and mathematical algorithms. Architects therefore strive (sometimes successfully) to give material form to the ideas of the mathematicians, thus generating new shapes.

Escher's theory on the regular division of the plane was the first step towards broader research, aimed at investigating the structure of the mental space of our imagination. It is no surprise that one of Escher's main sources of inspiration is the geometry of the Alhambra arabesques, which can be extended ad infinitum.

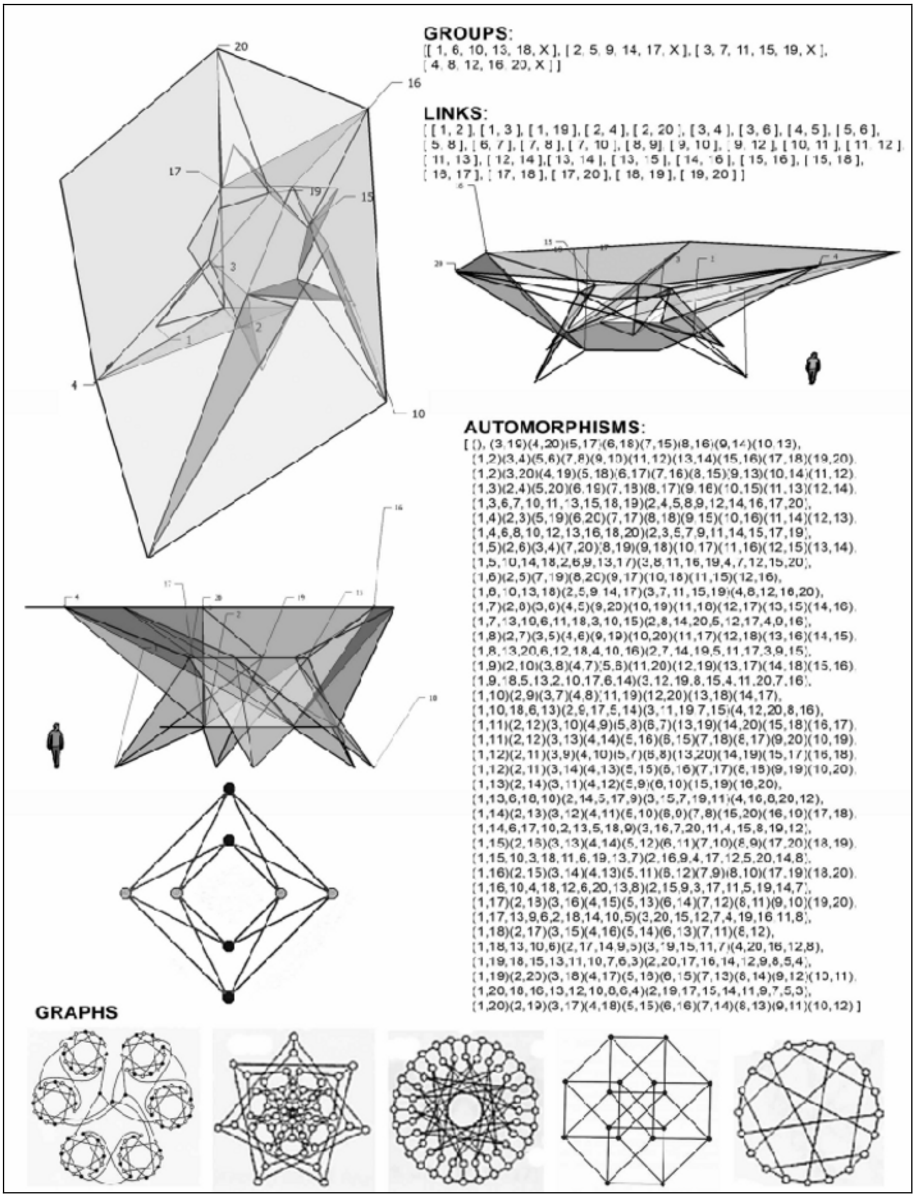

Fig. 2

Recent studies by Peter J. Lu and Paul Steinhardt of Harvard University [Lu and Steinhardt 2007] seem to demonstrate that aperiodic tessellation was already known in ancient times. In the mosaics of the mosque of Darb-i Imam in Iran (1453) never repetitive patterns (*girih*) are present: they are made from four or five different tiles (decagon, pentagon, rhombus, hexagon, triangle), joined to form decorations that are different from the traditional ones. Their creation relies on the same mathematical principles governing aperiodic Penrose tilings.

Girih and arabesques, *kolam* and Celtic knots, and rose windows are two-dimensional structures, but they sometimes evoke the planar projection of spatial geometries. Regular patterns similar to these geometrical structures can be found in graph theory and in permutation groups.

Experimenting with modern mathematical models

The architectural examples mentioned above almost seem to satisfy completely the many opportunities offered by the "classical" (linear, Euclidean and non-Euclidean) geometries, so we tried to make use of other geometries in order to obtain new "exotic", "symmetry effects".

We tried to test the use of some algebraic structures, such as group theory and graph theory, with the aim of building structural and architectural grids.

In order to obtain new types of "symmetries" (i.e., partial automorphisms), we started by constructing (according to the second part of Klein's Erlangen program [Klein 1893]) a geometry with a given group of automorphisms (fig. 2). We are trying to apply this idea to construct regular connected graphs (according the idea of "free mobility" due to Sophus Lie) that can be read as partial plans for a building.

There are a lot of algebraic and combinatorial results of graph and group theory that are suitable for creating and examining such graphs. As a first experiment, for example, we can use one of Frucht's results [Frucht 1949] to obtain, for each abstract group G, a regular graph of degree three having G as a group of automorphisms; this can be useful for easily obtaining (by only fixing a suitable set of points on the ground) many isostatic skeletons. Obviously, by filtering Cayley graphs, we can obtain other types of skeletons that may be interesting: the structure of the involved group G seems to produce useful suggestions for setting in the space the skeleton points, mainly when G is a product of (or generated by) two, three or four of its subgroups. Maybe we can obtain nice planar regular graphs suitable for building something such as geodetic domes or at least regular balls (with a hard center). Similarly we can try to get Hamiltonian graphs to be transformed in the skeleton of a tensegrity structure taking a cycle as a set of tie-beam.

Conclusion

Therefore what we may see is how much algebraic studies are involved in development of both non-Euclidean geometries and topological architecture. A continuous *fil rouge* ties polyhedra, kaleidoscopes, and Fuller's geodesic inventions to the formal research of some modern artists who are inquiring into n-dimensional spaces, such as Attilio Pierelli and Anish Kapoor.

In addition to his work on jewels and sculptures, Attilio Pierelli conceived a four-dimensional church for Santa Maria del Rosario in Viterbo (Italy) and the model, inspired by the hypercube, has been accepted by the bishop (fig. 3). At this writing, however, it is still in the design phase.

Admiring the Cloud Gate, also known as "the Bean" (2005), at the Millennium Park in Chicago, designed by Anish Kapoor, is a really interesting experience: under "the Bean", there are so many reflections that they can hardly be counted (fig. 4).

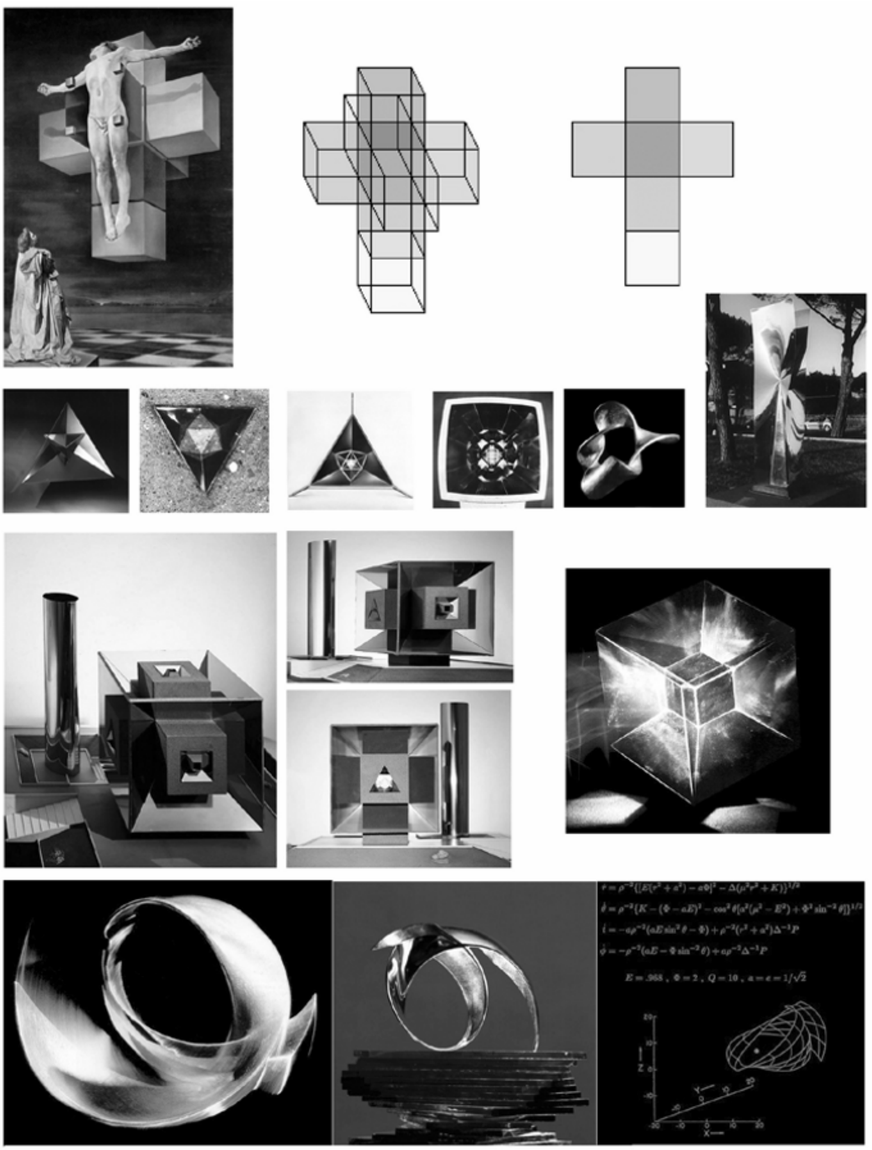

Fig. 3.

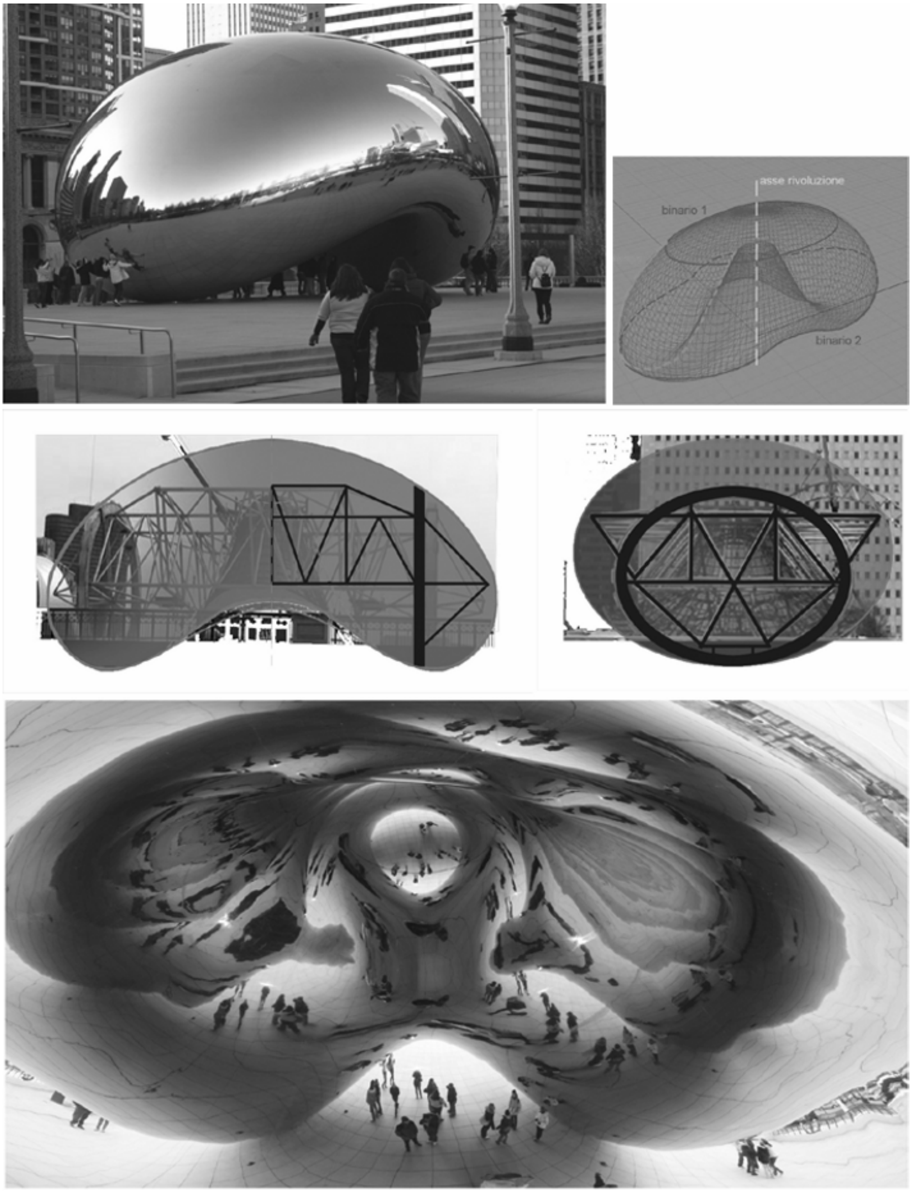

Fig. 4.

Further, the surrounding buildings are mirrored on the inside surface, but everything is deformed and altered, moving and changing with the spectator's own movement; the straight line obviously becomes a curve, but sometimes it changes direction.

It is not easy to replicate the peculiar reflections on the Cloud Gate's continuous surface, not even with a simplified and controlled 3D model. However architecture has never before witnessed anything as close to an n-dimensional space as this.

Notes

1. In 1996 Coxeter published a critic of Escher "*Circle Limit III*" where he demonstrated through trigonometry the mathematical precision of the work.
2. So called in honour of Buckminster Fuller, fullerenes earned their discoverers, Harold Kroto, Robert Curl and Richard Smalley, the 1996 Nobel prize for chemistry. Spherical fullerenes are called buckyballs, while cylindrical ones are called buckytubes.
3. Also called deconstruction, it is a postmodern movement in architecture. Decontructivist architects are F. O. Gehry, D. Libeskind, R. Koolhaas, P. Eisenman, Z. M. Hadid, the group Coop Himmelblau and B. Tschumi.

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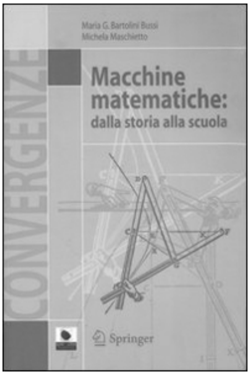
Didactics

Mathematical Machines: A Laboratory for Mathematics

Abstract. *Macchine matematiche; dalla storia alla scuola* is a book by Maria Grazia Bartolini Bussi and Michela Maschietto that provides a description of the reconstruction of many historical machines for drawing, and their impact on the history of mathematics. Underlying the book's thesis are ideas about education and developments in neuroscience. Laura Tedeschini Lalli discussed the book and the educational experiments and their goals with author Maria Grazia Bartolini Bussi.

Introduction



The Festival of Mathematics, which has taken place at Rome's Auditorium Parco della Musica for the past three years (this year's edition took place 19-22 March 2009), draws many visitors of all ages from all over. This year I met many architects there who came partly out of general curiosity, but also to see the exhibit entitled "Mathematical Machines" curated by the research group of the same name from the University of Modena and Reggio Emilia, coordinated by Mariolina Bartolini Bussi. The Roman architects were fascinated by the possibility of seeing up close how the mathematical machines work; the exhibit featured instruments designed centuries ago, reconstructed down to the smallest detail, and workable.

The book *Macchine matematiche: dalla storia alla scuola* (Milan, Springer Italia, 2006, 159 pp.), one of the books in the series entitled "Convergenze" published by the Unione Matematica Italiana through Springer Italia, provides a description of many of the reconstructed machines used for drawing, and their impact on the history of mathematics. The first half is dedicated to the instruments and the mathematical laws behind them, and gives the formulations of some of the "geometric problems" that they solve graphically. All of this is presented together with historical notes. The second half discusses educational experiments performed in schools on different continents, intended to help build the capacity for abstraction through knowledgeable use of the drawing machines.

The lengthy bibliography is divided according to topic, and distinguishes historical sources from modern works about history. It also includes educational experiments and works on teaching mathematics and the capacities for abstraction.

The book comes with a CD that contains animations of the drawings. It also contains other interesting documents, such as the policy of the Unione Matematica Italiana regarding a laboratory of mathematics in schools, whether such a laboratory is possible and its nature.

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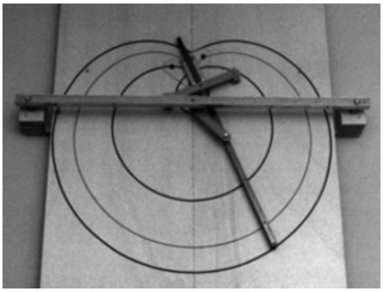


Fig. 1. Generation of fourth-degree algebraic curves, Pascals' snails

The historical approach is very nice, for non-mathematicians as well as mathematicians, with an entire chapter dedicated to a detailed comparison of Desargues and Descartes and their quite different models of rationality. The historical approach also makes it possible to look at modernity and the future, telling how branches of mathematics developed and were generalised from the line of reasoning that grew up around the mathematical machines for drawing, branches that are still fertile and today are abstract, elegant and apparently disconnected from the need for graphic representation.

I interviewed Mariolina Bartolini Bussi to learn more about her experiences with and thoughts about the machines and, more generally, about teaching mathematics.

Laura Tedeschini Lalli: The methods and aims of this research are typical of the Italian outlook; they are historical studies intended to regain possession of methods of teaching and learning.

Maria Grazia Bartolini Bussi: The project about machines was born of our interest in history, but from the very beginning it had an educational purpose, because the machines were to be used by students. This is different from what is usually done with historical or museum projectcs. For example, in the book we mention the 2001 exhibit "Nel segno di Masaccio" at the Uffizi in Florence, which featured original drawings and valuable instruments designed by engineers and built by specialised craftsmen. But these recreations were so precious that no one could touch them, and as a final result of a research project this was inconceivable for us. The machines had to be usable in order for them to be understood.

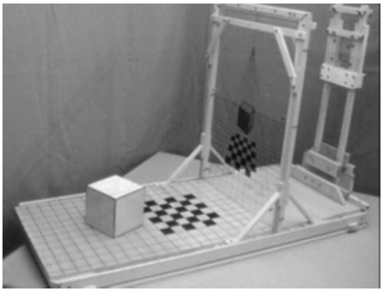
TL: Wasn't there anyone at the exhibit who was authorised to use the machines?

BB: No, not even the guides, who were only there to make sure that the visitors didn't touch them. I myself had used gestures to explain to my companions how the machines worked without touching them; the guides came up to listen, astounded at how much I knew about the machines, so I explained that we had a lot more machines like these, and that ours could be used, and so I knew them quite well. As I said, in the beginning we were interested in the historical value of the machines, but also in their educational value because one of our questions is, how can we communicate to students who live in a world that is so virtual, instantaneous and disconnected from history that they too are part of the unfolding of history? It might be possible to do this if they can use objects that themselves have a history. There is an educational message in our use of history: be proud of your culture because, for example, Italy has produced this kind of thing. Today we hear a lot about intercultural problems, but in order to speak about interculturalism we need to know what a culture is and love our own. Then later, loving our own culture, we can also open up to the world. I say this as someone who trains teachers, and I speak from having worked with and thought about these problems for years. So it's important to me to underline the epistemological aspect in the fullest sense of the term, that is, the epistemology of the building up of knowledge, even today. There is a cognitive aspect here that is quite strong, and harmonises with the results of recent research on the powerful physical aspect of experience, the fact that knowledge, even abstract knowledge, is embodied. This has also been confirmed by neuroscientists.

TL: This concept is quite explicitly expressed in your book as well. Can you tell us more about the results of the research of neuroscientists regarding abstraction?

BB: We actually maintain a certain distance from this aspect: research undertaken by very capable cognitive linguists in San Diego has unfortunately neglected the instrumental aspect. To me the subject is not a naked man facing the world; the subject is a man who lives in a culture that has produced artifacts, and the knowledge that has been built up is mediated by these artifacts. This does not appear in the analyses performed by cognitive linguists. The only artifact that they have taken into consideration is the one that, of course, they cannot exclude, which is language. Instead, we want the material aspect to be considered as well. If we take the next step and move to neuroscience, then I'll only note that the time required and complexity of the processes for learning mathematics are still too far removed from those studied using methods of neuroimaging. As far as materiality is concerned, you and I share this view, don't we?

TL: Yes, I teach mathematics in a school of architecture, and I know that the architecture students are skilful at thinking with pencils in hand. One of the studies that interests me greatly is the role of the physical-mathematical model in reinforcing a cultural formulation. This is difficult to discuss, because so often it leads to misunderstandings. Sometimes the physical-mathematical model is implied, and appears to be a in tool for measuring, because it underlies and governs the design of the tool. I would imagine that it is similar for instruments for drawing.



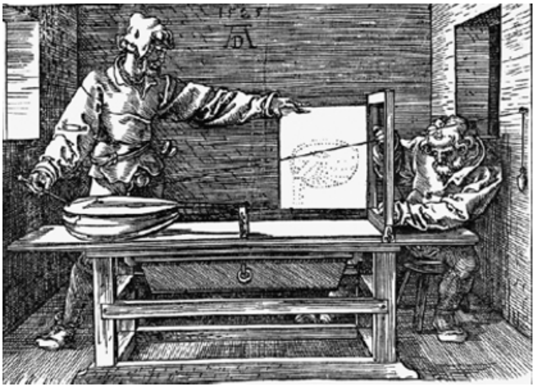


Fig. 2. above) Dürer's perspectograph, the "veil"; right) Albrecht Dürer drawing on glass (c.1520), etching

BB: When we talk about models we always have to add an adjective, that is, sometimes we mean mathematical models, sometimes physical models. Since we are involved with mathematical machines, we naturally speak of "models" in both senses: we have the mathematical model, and we also have a physical model of the mathematical model, that is, of the law that underlies it. We believe in this cultural approach, because it comes from history, and adds value to the aspects of mathematics that are tied to other aspects of human knowledge in history. We also make models, but this is a complementary approach. Sometimes the mathematical machines intentionally demonstrate mathematical laws that are already known (such as in the case of many of the machines for drawing curves).

Sometimes they precede the formulation of mathematical laws (as in the case of perspectographs).

TL: It seems to me that it is important to bring to conscious awareness the existence of an implied model. I see that you too share this conviction, because in the book you discuss experiments that you call “black box experiments”, which I also make with my students. I call them “inverse problems”, although perhaps this is too strong a term, because they are actually an initiation into the class of inverse problems. You give two drawn versions of the same figure, and ask which mathematical machine, or which procedure, changed one into the other. Asking which machine is used to create the representation already sets the stage for recognition of the model.

BB: It’s true. We are a group of researchers working in the teaching of mathematics, and thus we study what effects the intentional use of these mathematical machines have on processes of teaching-learning. We began in schools at the pre-university level, although more recently we have also begun to transpose the results to the university level, for teacher training for elementary schools and secondary schools.

TL: Many architects have studied theories and rules for graphic representation that were born around these machines without, however, actually using them. In your book you mention the collection at Cornell University. Do they allow their machines to be used like you do?

BB: Cornell has the collection of Reuleaux (1829-1905), who had reconstructed more than 800 fundamental mechanisms, the simple machines. They have a very interesting virtual library (http://kmoddl.library.cornell.edu/index.php). They also have there an experimental project: Daina Taimina, a Latvian who lives there, presents the machines that most lend themselves to manipulation at schools in the area. This kind of approach is related to the idea of a laboratory of mathematics as a laboratory for experimentation. Mathematics too has an experimental component which favours it most creative aspects, that is, the genesis of conjecture, and construction. In our schools, above all when dealing with something in three dimensions, everything is flattened in order to replicate the proofs that the teacher has shown on the blackboard or are shown in the textbooks. Never, or hardly ever, do we see the moment of discovery or invention. In one of the courses for teachers that I teach at the university, I spend a lot of time on this creative aspect. Take the machines, for example: some have furnished the basis for the lines of reasoning of geometers of the 1600s who did not have continuum theory at their disposal; so what was a continuum for them at that time? It was what could be described by motion. In spite of this they invented valid theorems and formulated proofs which, according to the standards of rigour of the times, were rigorous. Today we have dynamic geometry software which they didn’t have and which, at least in the plane, makes things easier. We recuperate the aspects of thought that are creative, inventive, and which otherwise schools would tend to neglect: the genesis of predictive hypotheses (how the machine will work); of interpretative hypotheses (why does a given machine work that way?). We do this work both when the machines are present and when they are not, that is, with the help of mental experiments. We think that these things can and must be done from the time that the students are very young, and we have had very positive responses even at the elementary school level. But if the teachers don’t themselves have experience with these processes, then only rarely are they able to propose this kind of experiment. In our elementary schools the students arrive at *reductio ad absurdum* proofs in a completely natural way by making objects move (mentally, without touching them) and they can see that these objects cannot move, because that would be contrary to the rules that they themselves discovered and memorised earlier. It is not the impossibility of motion that they discover, but the theoretical impossibility of motion. This is the most interesting aspect of our experiments: we have proven that this can be done with children eight, nine, ten years old, in the usual time required for learning. Everything is done through touch, and above all through the intelligent use of language, other semiotic systems, signs, drawings, gestures, words, reasoning. In the book we deal less with the educational results: it is a mathematics book, and thus it deals not only with the aspect of education but with those of history and mathematics as well. We were asked to write a "small" book, not an encyclopedic treatise, one that would be of interest to a diverse group of readers, but above all to teachers.

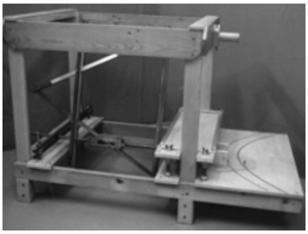


Fig. 3. Descartes's machine for hyperbolic lenses

TL: On the other hand, representation by means of manipulation and abstraction is interesting to a wide range of people. How has it been working with your experiments in places that are far apart both culturally and geographically? I like to say that "our eyes are culturally trained"; all of us, even those from other countries who come to Italy to study, have eyes that are culturally trained by the same visual culture. Part of this visual culture, for example, is one-point perspective. When eyes are not culturally trained with regards to the same artifact, what happens?

BB: We hardly ever perform the experiments directly ourselves. They are usually performed by our colleagues who live there with whom we have collaborated. In some case doctoral theses are involved; sometimes our experimental protocol has been modified. These activities have generally worked very well everywhere. We have different forms of collaboration on all five continents. Sometimes our colleagues have begun their own work, then learned about ours and contacted us, leading to us working together, especially on articulated systems (machines to lay out curves and pantographs), that is, on the more technological aspects. We haven't always pursued the same objectives, and we have tried, more briefly in the book and more in-depth in the CD, to report on the experiments. Naturally, the aspect of perspective is much more typically Italian, closer to our own culture.

TL: Even closer to Italian culture than to European culture?!

BB: As far as perspective is concerned, not even in Europe have we found researchers to collaborate on experiments with us: the few experiments on perspective that have been done, we have done ourselves. It might be that such experiments are made in schools of architecture, but we are not aware of any. Others have worked on articulated systems, on machines to lay out curves, pantographs, machines for transformations: these are somehow universal. I personally followed some phases of experiment in Mexico, but this was a school based on a European system and the instructors had in any case earned their Ph.Ds in Europe. The instructors in Mali and Mexico that we talk about in the book earned their degrees in Grenoble. So in some way in order to be part of a project like ours the researcher has a kind of cultural conditioning; the students may not, but the researcher does. In other words, we have had a good response from schools that can be defined as Western, even though they might be in Africa: they are schools in any case that follow a European model. Many wonderful experiments have been performed in Australia, including experiements with students who are quite young.

TL: Have you come across cultural differences in all your long experience?

BB: Without a doubt, cultural differences exist and become evident. Rarely do they emerge when using an artifact, a machine: an artifact already embodies knowledge. More often they come out when natural phenomena are concerned, for example, shadows. We have worked a great deal with shadows from the sun; some of our machines allude to shadows, they are models (physical models of mathematical models) of shadows, but they deal with the mathematical model, built with our Western mentality. We have had collaborators who have performed experiments in African, in Italian schools in Ethiopia and Eritrea. Even though they go to an Italian school, the children are not Italian, and they have perceptions and a system of beliefs about shadows that are very different from those of Italian children. The shadow is a very thorny object, above all when the shadow of one's own body is involved, because it is our "double". As long as we study shadows of sticks and neutral objects, all is well, or at least, it is easier to construct the mathematical object. Of course, the cultural aspect has to be taken into account. We are being pressed to translate this book, most likely into English. I don't think, however, that this book is universal. I am a member of the International Commission for mathematics teaching and I assure you that we come up against cultural differences as well as differences in models of rationality. Mathematics is not "simply" universal, as many like to think. Or better, mathematics as a crystallised product can be universal, but the process of approaching mathematics might not be the same in all cultures.

TL: Can you tell us more about cultural differences and convergences and mathematics?

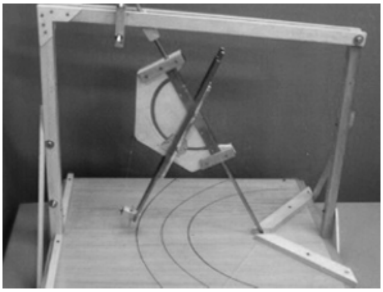


Fig. 4. A perfect compass

BB: One of my Japanese colleagues worked on antique Japanese books and found articulated systems very similar to ours. They are the same as our instruments for laying out continuous curves from the 1600s. One article written by six scholars, including a Japanese and a Latvian, also retraced the European influence both in the East, in Japan, and in the West, in the United States, using the emblematic case that we cited of the Cornell collection.

However, in ancient Chinese technology there were machines that were extremely similar to ours, which they used for things that are extraordinary to us, such as realising etchings on jade.There are aspects of technology that are shared by many countries.Instead, we have a situation which is rather exceptional where perspective is concerned. The birth of perspective as a science is rather recent, as far as we know, and so we can follow it from its inception. Straightedge and compass were born somewhere at the dawn of time, that is, we know very little about what geometry was before Euclid, but on the other hand it is possible to reconstruct the birth of perspective. In this case, technology – the machines – came before the mathematics, because the theory of transformations and projective geometry developed later. Many mathematicians think of mathematics as the queen of sciences: that pure mathematics comes first and applied mathematics only later. Instead, quite often fundamental theories of mathematics, such as projective geometry, were born from applications. We have many examples to counter this widespread belief. So, we can use the machines to break the stereotype: mathematics is only a product of man's culture, which has dialectic relationships with the other products, before, after and sometimes at the same time; sometimes it is separate. We get the impression of a history of mathematics that is sometimes too cut and dried. History can help us quite a lot to see the world and the evolution of the building up of knowledge. If we read some of the authors of the 1600s, such as Desargues, we come to see that the way of thinking is not that of Descartes, who is considered today by many to be *the* model for rational thought. The complex aspects of our history can help us to accept the diverse models of rationality that have contributed to the construction of mathematical meanings, even at the cost of renouncing rigour. René Thom, winner of the Fields Medal, once said that if he had to choose between meaning and rigour, he would unhesitatingly choose meaning. If we were to interview the great mathematicians of today about their creative processes, they would confess that in reality the creative process isn't at all a deductive, rigorous process: it goes forward by intuition, which is then adjusted as it goes along. Only the mathematicians with more modest gifts work deductively.

TL: Can we say that, in formulating hypotheses, you emphasise the inductive moment?

BB: In addition to the inductive process, in many cases there is another process, described by Peirce, that of abduction: recognising something that you might have already seen and being able to connect it to other systems of knowledge, being able to make a conjecture which has still to be proven, but which already has a higher degree of probability of being valid. I sometimes say to my university students, "Look at this figure: do you recognise something?" They might close their eyes, in order to see it better in their minds, then suddenly they open them again and begin to laugh: they recognised something! This is an abduction, recognising in this complex figure something that they already knew. It is clear that the more things you know, the easier it is to make abductions. If you know only straight lines and conics, then you see an arch and either think of a straight line or a conic, or nothing at all comes to mind. This year we looked at some third- and fourth-degree curves, studying their mechanical generation, and they are now able to recognise those too, to produce hypotheses. So they have increased their creative potential, because knowing more things, they are able to recognise, in complicated situations, more things that are related to the objects that they have constructed. Creativity is also due to the quantity of knowledge that you have internalised: the more you know, the more creative you are. The more things you know, the more things you can be creative about, carrying them from one area to another. So, in addition to deduction and induction, there is also abduction.

TL: What were your experiences with a public that is adult and different from that of the schools, and with architects in particular? They find this idea of reconstructing machines to be particularly interesting.

BB: My impression, and this was confirmed at the Festival of Mathematics in Rome, is that everyone sees something different. One can see concretely the objects that have been studied; there is probably an aesthetic aspect as well, because our machines are beautiful. There is the potential to interest a quite diverse public. In Modena we had a seminar with those interested in philosophy and letters. However, I see that the architects, who up until now we have not worked with a great deal, might be or should be among our favourite interlocutors. Alessandra Mariotti and I have been in contact with Gabriela Goldschmidt, an Israeli architect who was taking a sabbatical year at MIT. On that occasion we were invited to MIT, because she had so appreciated these machines. They had briefly involved us in an international group working on cognitive processes in design, which was very interesting. They studied the processes used by architects to design and create new projects; it was tied to the study of creativity. We saw how they worked, I would call it cognitive architecture: they collected notebooks, conducted interviews, showed architects who reasoned out loud, in order to understand how a design is put together, what ways and means are used. We found it interesting that there were aspects that we also find in the mathematical way of thinking: a continual oscillation between overall intentionality and a return to very small, almost microscopic aspects of design. But the experience lasted only a short time. I remember that the dean of the school of architecture at MIT didn't miss a word, in the end.

Translated from the Italian by Kim Williams

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The images all come from the website of the Laboratorio Macchine Matematiche of the Università di Modena e Reggio Emilia.
http://www.museo.unimo.it/theatrum/macchine/_00lab.htm

About the author

Laura Tedeschini Lalli, Ph.D., is full professor of Mathematical Physics in the Facoltà di Architettura of Università Roma Tre. Her formal training is in mathematics and in musical composition. She received her Ph.D. in applied math from the University of Maryland in College Park. Her research is in chaotic dynamical systems (theoretical) and some applications; in sonification of data; in the (often implicit) role of the mathematical model in the communication of science. She is among the founders of European Women in Mathematics (EWM). She coordinates the Laboratorio di Matematica di Roma Tre, where architects and mathematicians work together.

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Conference Report

Fortification in Focus - Mathematical Methods in Military Architecture of the 16th and 17th Centuries and their Sublimation in Civil Architecture

3-5 October 2008, Dresden

In recent years the importance of mathematics as a basic discipline not only for the sciences but also for the humanities has been rediscovered and become the subject of interdisciplinary investigations. Two examples among many are the publication *Die mathematischen Wurzeln der Kultur. Mathematische Innovationen und ihre kulturellen Folgen* (The mathematical roots of culture – mathematical innovations and their cultural consequences) edited by Jochen Brüning and Eberhard Knobloch, Munich 2005, and the conference "Was zählt. Präsenz und Ordnungsangebote von Zahlen im Mittelalter" (What counts. The presence and medial function of number in the Middle Ages) held 2006 in Berlin. The specific role of geometry in the field of landscape architecture has been pointed out lately and rather pertinently by Volker Remmert in his article "'Il faut etre un peu geometre'- Die mathematischen Wissenschaften in der Gartenkunst der Frühen Neuzeit" (One has to be a bit of a geometer – the mathematical sciences in the art of the garden in early modern age), published in the catalogue of the exhibition "Wunder und Wissenschaft – Salomon de Caus und die Automatenkunst in Gärten um 1600", Düsseldorf 2008, (Miracle and science – Salomon de Caus and the art of automatons in gardens of the 1600s).

"The principal aim of mathematics is not application, but the creation of culture" – the idea embodied in this statement by mathematician Gerhard Frey can be traced back to the beginnings of human history. From the times of the Babylonians and Egyptians, the discovery, the knowledge and the application of mathematics to daily life was the driving force behind all technical innovations of all cultures. One significant example of this development was the changes of fortification systems around 1500, brought about the fall of Constantinople as a result of the hail of bullets fired by the Ottomans in 1453, which made it obvious that city walls built in the Middle Ages were no longer strong enough to

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provide protection from artillery fire. Three cultural factors strongly influenced the new developments. First, the Gutenberg press was invented, and the printing of books on paper facilitated the circulation of knowledge. Euclid's *Elements* was the first book to be published and widely available beyond the confines of monastery libraries and the few private libraries of privileged scholars. The knowledge of geometry, up to then taught as part of the *artes liberales* at the universities, became widespread, with far-reaching consequences for cultural developments. Second, the invention and development of perspective drawings during the fifteenth century created a new visual awareness of space and its representation. Finally, the rediscovery of Platonic philosophy appears to have to compelled the introduction and transmission of geometrical forms into applied arts and architecture.

The idea for the "Fortifications in Focus" conference came from the meeting of Bettina Marten, Michael Korey and Ulrich Reinisch, who were all working on fortifications but from different points of view: Bettina Marten, an art historian, focuses her investigations on the relationship between mathematical knowledge and its influence on military architecture; Michael Korey, a mathematician, is interested in the instruments of measurement and their meaning within the courtly circles; Ulrich Reinisch, also an art historian, is investigating the transmission of elements and methods appertaining to military architecture into civil architecture and landscape architecture. The goal of the conference, to which were invited historians of science, art, and architecture, as well as experts in fortification, was to illuminate aspects of mathematics during the period in question and the use of mathematical instruments, as well as the concomitant military, visual, and rhetorical claims in fortification and related architectural fields.

In their introductions, the organizers explained the fundamental ideas underlying their interdisciplinary approaches. These were then expanded by the speakers who followed. Bruno Klein (TU Dresden) addressed the problems of the history of the visibility and non-visibility of military architecture and formulated on this basis a program to investigate the subject of the meeting within the history of culture and science. Bettina Marten (TU Dresden) outlined the many and diverse interdisciplinary starting points connected with the topic of military architecture, including the history of mathematics, of architecture and of the theory of architecture, of military history, of science and of social science; all of these make evident the far-reaching relevance of fortification. Stefan Bürger (TU Dresden) concentrated his observations on the heterogeneous understanding of theory and practice and the divergent positions of the engineers and theorists caused by diverse methods in the field of fortification building; these differences already existed in the sixteenth century, and formed the basis for the divergences today. On the basis of the psychoanalysis of Sigmund Freud, Ulrich Reinisch (Humboldt-University Berlin) reflected on the terms "anxiety" and "sublimation" which dominated the conditions (of anxiety) in the Middle Ages and how these were overcome by intellectualizing them as methods of defensive action, which launched the changes of the defensive systems in the 1500s. Michael Korey (Mathematic-Physical Salon of the State Collections of Art Dresden) referred to the role of the courtly collections of scientific instruments and libraries. The collections of contemporary treatises on measurement and fortification architecture constituted centres of innovative intellectual activity within the European aristocracy; shared knowledge was supported by the exchange of precisely-worked and lavishly decorated instruments as gifts.

The precise categorizations and descriptions of the complicated and widely branching system of Early Renaissance knowledge led to military architecture being classified as an operative art between mathematics as *scientia contemplativa* and crafts as *ars mechanica*. Geometry functioned as a connecting link because of its practical application in the field of measurement on the one hand and through literary tradition on the other hand, both as a method of cognition of nature and as a foundation for human activity (Orietta Pedemonte, Genova). This is the explanation of the analogies established during the sixteenth century between military architecture and machine construction, between military architecture and civil architecture. The basis of all these areas is operative or practical geometry, which had been mastered by both military engineers and architects (Tobias Büchi, Zürich). Because of this, the transmission into civil and landscape architecture seems to be a natural consequence, because these used the methods and artistic elements developed in military architecture, transferring the specific symbolic elements of strength and power to civil architecture (Christof Baier, Marion Hilliges, Judith Schlereth, all Berlin). The mastery of geometry as part of speculative science as well as part of the *artes mechanicae* made it possible for artists like Leonardo da Vinci, Albrecht Dürer and Michelangelo Buonarotti to deal with apparently incompatible disciplines such as the arts, military architecture and machine-building. Similar to Albrecht Dürer, Leonardo da Vinci was convinced that the perfect design for a fortress respected the ideal form of the circle, a form he also used for mobile artillery weapons, a prototype of modern tanks (Kim Williams, Torino).

Of fundamental importance in the process of designing are the visual representations, first in perspective drawings and second in three-dimensional models. Whereas the models were built to show volumes, the function of the different parts of the fortifications, and their integration into the surrounding topography, the drawings developed more and more into the preferred medium for representation, because, taking advantage of a bird's eye view, they provided the best picture of the complexity of the fortresses and their extension into the surroundings. A new awareness of space and territory was thus created. Perspective drawing as a medium was used as an instrument for presenting the functional aspects of the bastioned fortification system in a more credible way, because the applied "objective" mathematical methods were more suitable than free-hand visualizations of earthworks, which were practice orientated and of rounder shapes (Stefan Hoppe, Köln; Ralf Gebuhr, Cottbus). The radical change in military architecture has also been connected with the changes in the process of designing, which came to include the moment of movement as well: the design lines were contemporaneously lines of movement because they were identified as lines of fire, as the analyses of Galileo Galilei´s treatise of fortification (1592/93) have shown (Horst Bredekamp, Berlin). The search for security within the idea of order in geometrical form was influenced, on one hand, by the rediscovery of the sixth book of the *Historíai* written by Greek historian Polybius and, on the other hand, by the acceptance of geometry as the tool for objective scientific discoveries of natural laws, which was understood to be and taught as a secure foundation (Nicola Aricò, Messina; Bernd Roeck, Zürich; Jeroen Goudeau, Deventer). The range of the intellectual performance and the authority of mathematical methods related to fortifications in regard to social relevance and importance can be demonstrated by comparison with other arts deemed mathematical, such as music or sundialing, in view of the relationships between mathematicians, practitioners, artisans and the public (Stephen Johnston, Oxford).

Aristocratic education was founded on the supposedly unalterable knowledge of mathematics and fortifications. The sons of European courts acquired their knowledge at the hands of tutors who were experts of mathematics and fortification, such as Galileo Galilei for the Medici family or Carlo Thetis for the Dukes of Saxony. Among the materials used for teaching were treatises containing theoretical and practical instructions, such as, for example, Girolamo Cataneo´s *Arte de militare* or *Arte de misurare* (Pascal Brioist, Tours). The members of the aristocracy were also military commanders, who were personally involved in battles. One of their most important weapons was the dagger, which could be transformed into a compass, the traditional allegorical symbol of geometry in the sixteenth century. The technological and cultural changes, in which the keys to political and military power were entrusted to geometry, implied an extraordinary convergence of technical and scientific capabilities ranging from perspective drawing to topographic surveying, from calculation to measurement (Filippo Camerota, Florence). The far-reaching consequences of these changes on the political reality were shown by the analyses of the system of fortifications built by Emperor Charles V in the southern parts of the old Netherlands. The fortified cities of Mariembourg, Hesdinfert, Charlesmont and Philippeville, founded between 1546-1560 on strategically important sites, all had polygonal layouts with radial or orthogonal street systems. The construction of these cities, under the direction of Italian or Dutch engineers, took many years and consumed significant sums of money (Bernhard Roosens, Gent/Berlin).

The meeting began and ended with excursions: The first was a guided tour to the remaining parts of the fortifications of Dresden at the Brühlsche Terrasse near the river Elbe, where Dr. Heiko Berger (Militärhistorisches Museum, Dresden) gave an introduction to the system of artillery weapons as well. The other excursion went to Theresienstadt in the Czech Republic. This fortified city, built 1780-1790 by Emperor Josef I, has survived almost intact, but has been stigmatized because it was used as a Jewish ghetto by the German Nazis from 1941 to 1945. Astrid Debold-Kritter, who has been investigating the original eighteenth-century plans for many years, explained the concept of the urban structure.

The “Fortifications in Focus” conference was organized by Dr. Bettina Marten (Institut für Kunst- und Musikwissenschaft der Technischen Universität Dresden), Dr. Michael Korey (Mathematisch-Physikalischer Salon der Staatlichen Kunstsammlungen Dresden) and Prof. Dr. Ulrich Reinisch (Kunstgeschichtliches Seminar der Humboldt Universität Berlin). Financial support was provided by the Gerda Henkel Stiftung, Düsseldorf, and the Gesellschaft der Freunde und Förderer der TU Dresden. The conference project was given an award in the competition “Kopf und Zahl” by the German Ministry of Education and Investigation, announced on occasion of the scientific year of mathematics. The papers that resulted from the presentations at the conference are now in preparation for publication, scheduled for the 2009.

About the author

Bettina Marten studied art history, archaeology, philosophy and social sciences at the University of Hamburg with professors Horst Bredekamp and Martin Warnke. She wrote her doctoral thesis on the fortifications built by Vespasiano Gonzaga for Philippe II in Spain and Italy. Her principal interests are urban development from the Middle Ages onwards, including contemporary movements; Spanish art history; processes of transculturation. She teaches at the Universities of Dresden and Frankfurt/Main.